

Day 13. 7/19 Monday

Functions

Finish pages 1-23 by 7/20

Functions In-Class Notes

Vertical Line test:

Terms:


- | | |
|------------------------|--|
| 1. Parallel lines | A. $f(-x) = -f(x)$ |
| 2. Perpendicular lines | B. lowest point in a section of a graph |
| 3. Odd function | C. x values |
| 4. Even function | D. the rate of change of a line |
| 5. Relative maximum | E. y values |
| 6. Relative minimum | F. highest point in a section of a graph |
| 7. Domain | G. lines with the same slope |
| 8. Range | H. uses () and/or [] to indicate domain and range |
| 9. Slope | I. $f(-x) = f(x)$ |
| 10. Function | J. lines with opposite reciprocal slopes |
| 11. Interval Notation | K. special relation where one input gives exactly one output |
| 12. x-intercept | L. crosses the x-axis |
| 13. y-intercept | M. crosses the y-axis |

Function Values

The equation $y=2x+3$ is written as $f(x)=2x+3$ in function notation.

The function value of f at $x=c$ is denoted as $f(c)$.

For instance, if $f(x)=5x-3$, $f(2)$ is the value of f at $x=2$ and $f(2)=5(2)-3=7$.

EXAMPLE 1  If $f(x) = 3x + 2$, find each value.


a. $f(-2)$


b. $-2[f(-1)] + f(-2)$

c. $f(x + h)$

d. $f(a + h)$

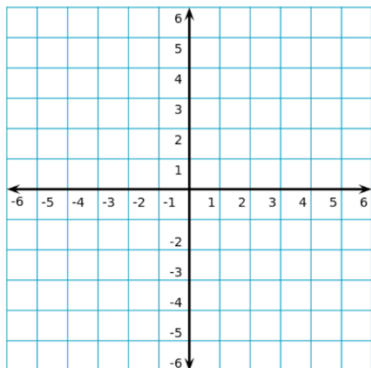
e. $f\left(-\frac{1}{2}x + 1\right)$

EXAMPLE 2  If $f(x) = f(3x) + 6$ and $f(6) = 8$, find $f(2)$.

EXAMPLE 2  If $f(x) = f(x + 3) + 6$ and $f(10) = -3$, find $f(7)$.

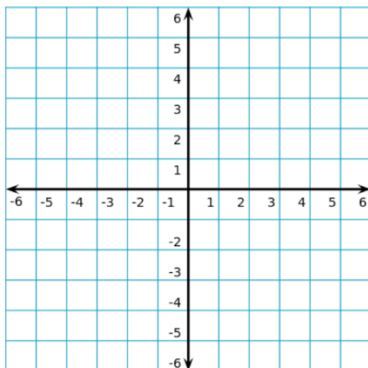
Types of functions

Constant Function: $f(x) = 2$



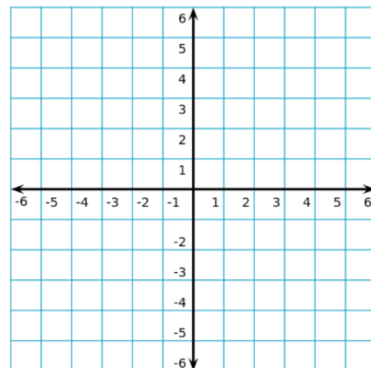
D: **R:**

Identity: $f(x) = x$



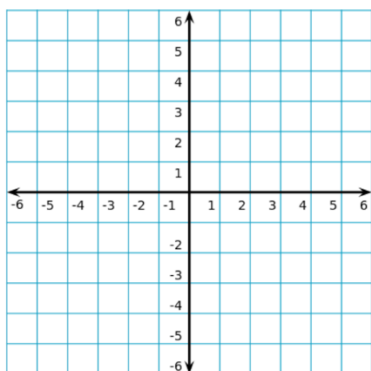
D: **R:**

Absolute Value: $f(x) = |x|$



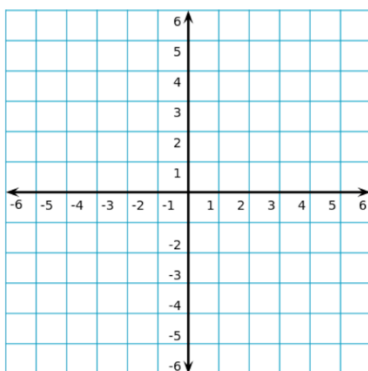
D: **R:**

Quadratic: $f(x) = x^2$



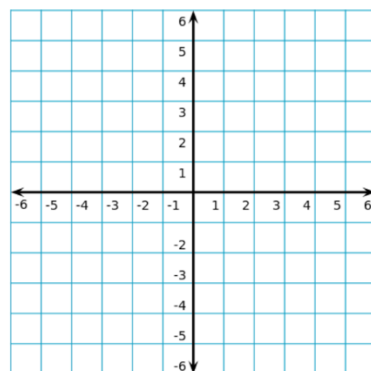
D: **R:**

Cubic: $f(x) = x^3$



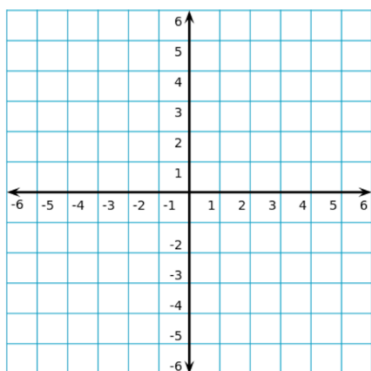
D: **R:**

Square Root: $f(x) = \sqrt{x}$



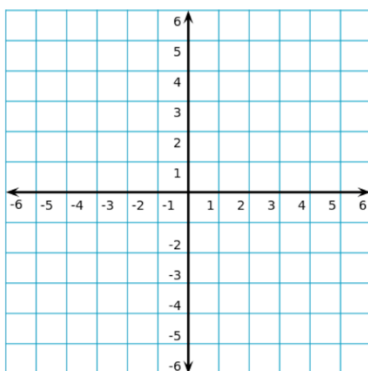
D: **R:**

Cube Root: $f(x) = \sqrt[3]{x}$



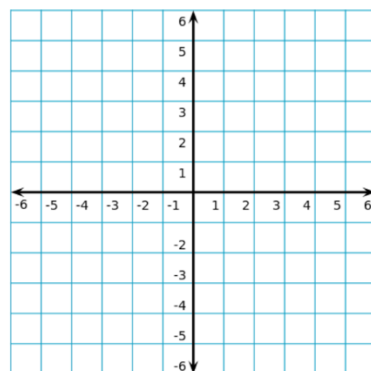
D: **R:**

Reciprocal: $f(x) = 1/x$



D: **R:**


Reciprocal Squared: $f(x) = 1/x^2$




D: **R:**

Domain of the square root function.

: When finding the domain of the square root function, set the term inside the square root greater than or equal to zero.


EXAMPLE 1  Find the domain of $f(x) = \sqrt{-x+18}$

EXAMPLE 2  Find the domain of $f(x) = \sqrt{x^2 - 4}$

Polynomial functions

End Behavior

	Even power	Odd power
+	$g(x) = 2x^2 - 5x + 3$	$g(x) = x^3 - 3x^2$
-	$f(x) = -x^4 + 2x^2 - 8$	$f(x) = -3x^3 + 5x$

EXAMPLE 3  Find all zeros of the polynomial function & graph.

$$f(x) = -x^5 + 5x^3 - 4x$$

$$f(x) = x^4 + 2x^2 - 8$$

EXAMPLE 4  Find a third degree polynomial with the following zeros: -3,4i

Functions - Homework

1

The graph of $f(x) = ax^3 + x^2 - 18x - 9$ intersects the x -axis at $(3,0)$. What is the value of a ?

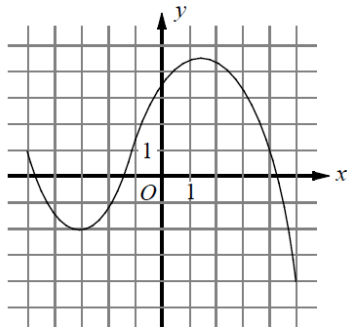
- A) -1
- B) 0
- C) 1
- D) 2

2

In the xy -plane, the graph of function f has x -intercepts at -7 , -5 , and 5 . Which of the following could define f ?

- A) $f(x) = (x-7)(x^2 - 25)$
- B) $f(x) = (x-7)(x^2 + 25)$
- C) $f(x) = (x+7)(x^2 - 25)$
- D) $f(x) = (x+7)(x^2 + 25)$

3

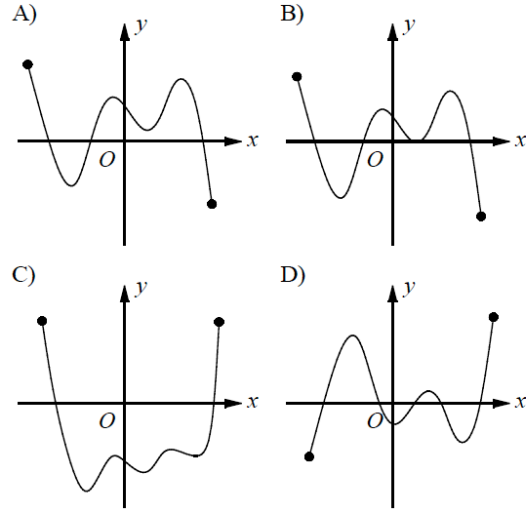


What is the minimum value of the function graphed on the xy -plane above, for $-5 \leq x \leq 5$?

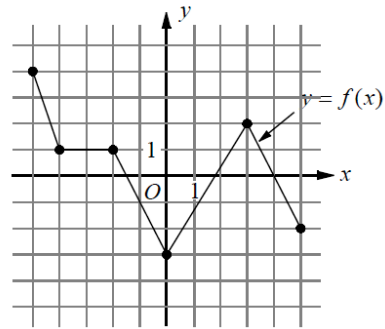
- A) -4
- B) -3
- C) -2
- D) $-\infty$

4

If function f has four distinct zeros, which of the following could represent the complete graph of f in the xy -plane?



5



The complete graph of function f is shown on the xy -plane above, for $-5 \leq x \leq 5$. Which of the following is/are true?

- I. f is strictly decreasing for $-5 < x < 0$.
- II. $f(-3) = 1$
- III. f is minimum at $x = 5$.

- A) I only
- B) II only
- C) III only
- D) I and II only


Remainder Theorem and Factor Theorem


Remainder Theorem


IF a polynomial $f(x)$ is divided by $x-c$, the remainder is $f(c)$.

Factor Theorem

The polynomial $f(x)$ has $x-c$ as a factor if and only if $f(c)=0$.

EXAMPLE 1  Find the remainder of $f(x) = x^3 + x^2 - 6x - 7$. Divided by $x + 2$.

EXAMPLE 2  Find the value of a if $f - 3$ is a factor of $f(x) = x^3 - 11x + a$.

EXAMPLE 3  Find the value of k if $f(x) = 3(x^2 + 3x - 4) - 8(x - k)$ is divided by x .

EXAMPLE 3  Find the x -intercept of f if $f(x) = 2x^2 + x - 10$.

Polynomial Functions Homework

1

If -1 and 1 are two real roots of the polynomial function $f(x) = ax^3 + bx^2 + cx + d$ and $(0, 3)$ is the y -intercept of graph of f , what is the value of b ?

- A) -3
- B) -1
- C) 2
- D) 4

2

What is the remainder of polynomial $p(x) = 81x^5 - 121x^3 - 36$ divided by $x + 1$?

- A) -76
- B) -36
- C) 4
- D) 6

3

If $x - 2$ is a factor of polynomial $p(x) = a(x^3 - 2x) + b(x^2 - 5)$, which of the following must be true?

- A) $a + b = 0$
- B) $2a - b = 0$
- C) $2a + b = 0$
- D) $4a - b = 0$

4

x	$f(x)$
-4	-10
-3	0
-1	-4
2	20

The function f is defined by a polynomial. Some values of x and $f(x)$ are shown in the table above. Which of the following must be a factor of $f(x)$?

- A) $x + 4$
- B) $x + 3$
- C) $x + 1$
- D) $x - 2$

5

$$x^3 - 8x^2 + 3x - 24 = 0$$

For what real value of x is the equation above true?

6

If $x > 0$, what is the solution to the equation $x^4 - 8x^2 = 9$?



Piecewise Functions - In Class Notes

EXAMPLE 1

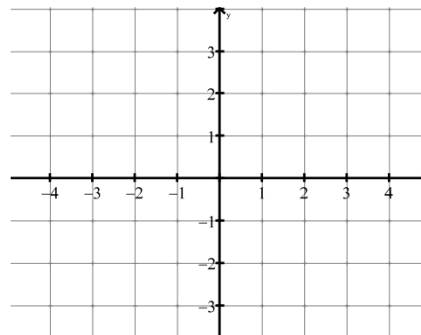
Sketch the graph of the function $g(x) = \begin{cases} -2x+3 & x < 1 \\ 3x-2 & x \geq 1 \end{cases}$, labeling at least 3

points with their coordinates.

What is the domain of this function? _____

What is the value of $g(-3)$? _____

What is the value of $g(4)$? _____



EXAMPLE 2

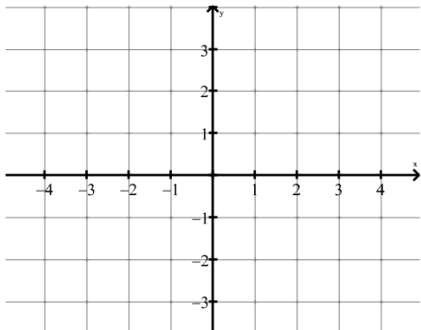
Sketch the graph of $h(x) = \begin{cases} |x| & -2 \leq x < 0 \\ 1 & x = 0 \\ x^3 & x > 0 \end{cases}$,

labeling at least 3 points with their coordinates.

What is the domain of this function? _____

What is the value of $h(-14)$? _____

What is the value of $h(6)$? _____



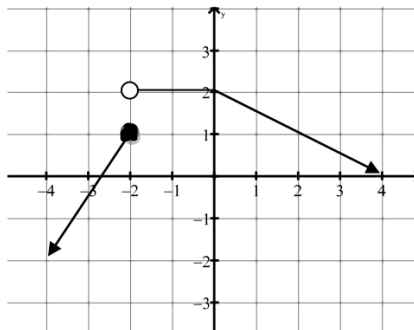
EXAMPLE 3

The graph of a piece-wise function is shown at right. Write the definition for the function.

$$f(x) = \left\{ \begin{array}{ll} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{array} \right\}$$

What is the domain? _____

What is the range? _____



Composite Functions - In Class Notes

Given the two functions f and g , the **composite function**, denoted by $f \circ g$, is defined as $(f \circ g)(x) = f \circ g(x) = f(g(x))$, read “ f of g of x .”

EXAMPLE 1

If $f(x) = x^2 + 1$ and $g(x) = x - 2$, find the following.

- a. $(f \circ g)(x)$ b. $(g \circ f)(x)$ c. $(f \circ g)(3)$

EXAMPLE 2

If $f = \{(-1, -3), (2, 5), (4, 1)\}$ and $g = \{(-2, 2), (1, 3), (6, -1)\}$, find the following.

- a. $(f \circ g)(-2)$ b. $(g \circ f)(4)$ c. $(f \circ g)(6)$

EXAMPLE 3

Let $f(x) = \sqrt{x^2 + 5}$, find $f \circ f \circ f(1)$.

Composite Functions - Homework

1

If $f(x) = x^2 - 3x - 1$ and $g(x) = 1 - x$, what is the value of $f \circ g(-2)$?

- A) -3
- B) -1
- C) 1
- D) 3

2

If $f = \{(-4, 12), (-2, 4), (2, 0), (3, \frac{3}{2})\}$ and $g = \{(-2, 5), (0, 1), (4, -7), (5, -9)\}$, what is the value of $g \circ f(2)$?

- A) -9
- B) -7
- C) 1
- D) 5

3

A function f satisfies $f(-1) = 8$ and $f(1) = -2$.
A function g satisfies $g(2) = 5$ and $g(-1) = 1$.
What is the value of $f(g(-1))$?

- A) -2
- B) 1
- C) 5
- D) 8

4

If $f(x) = \frac{1-5x}{2}$ and $g(x) = 2 - x$, what is the value of $f(g(3))$?

- A) -7
- B) -2
- C) 2
- D) 3

Questions 5 and 6 refer to the following information.

x	$f(x)$	$g(x)$
-2	-5	0
0	6	4
3	0	-5

The table above gives values of f and g at selected values of x .

5

What is the value of $f(g(-2))$?

6

What is the value of $g(f(3))$?



Recursive Formula - In Class Practice

A recursive formula for a sequence describes how to find the n th term from the term(s) before it.

EXAMPLE 1

A sequence is recursively defined by $a_n = a_{n-1} + \frac{2}{n}$. If $a_0 = 3$, what is the value of a_3 ?

EXAMPLE 2

For next year's vacation, Cabrera deposited \$2,000 into a savings account that pays 0.5% compounded monthly. In addition to this initial deposit, on the first day of each month, he deposits \$200 into the account. The amount of money n months after he opened the account can be calculated by the equation, $A_n = (1 + 0.005) \cdot A_{n-1} + 200$.

According to the formula, what will be the amount in Cabrera's savings account three months after he started it?

Recursive Formula - Homework

1

A sequence is recursively defined by

$a_n = \sqrt{(a_{n-1})^2 + 2}$. If $a_0 = \sqrt{2}$, what is the value of a_2 ?

- A) $\sqrt{5}$
- B) $\sqrt{6}$
- C) $\sqrt{8}$
- D) 3

2

A sequence is recursively defined by

$a_{n+1} = a_n - \frac{f(a_n)}{g(a_n)}$. If $a_0 = 1$, $f(x) = x^2 - 3x$, and $g(x) = 2x - 3$, what is the value of a_2 ?

- A) -3
- B) $-\frac{1}{5}$
- C) 2
- D) $\frac{3}{2}$

3

If $f(x) = \sqrt{2x^2 - 1}$, what is the value of $f \circ f \circ f(2)$?

- A) $\sqrt{10}$
- B) $\sqrt{15}$
- C) $\sqrt{21}$
- D) 5

4

If A_0 is the initial amount deposited into a savings account that earns at a fixed rate of r percent per year, and a constant amount of $12b$ is added to the account each year, then amount A_n of the savings n years after the initial deposit is made

is given by the equation $A_n = (1 + \frac{r}{100}) \cdot A_{n-1} + 12b$.

What is A_3 , the amount you have in the savings three years after you made the initial deposit, if $r = 5$, $A_0 = 12,000$, and $b = 400$?

- A) \$23,070.00
- B) \$26,048.00
- C) \$29,023.50
- D) \$35,274.68

5

The number of gallons, P_n , of a pollutant in a lake at the end of each month is given by the recursively defined formula $P_n = 0.85P_{n-1} + 20$.

If the initial amount P_0 of a pollutant in the lake is 400 gallons, what is P_3 , the amount of pollutant in the lake at the end of the third month, to the nearest gallon?

- A) 297
- B) 285
- C) 273
- D) 262



Inverse Functions - In Class Practice

- If $g(x)$ is the inverse function of $f(x)$, then $g \circ f = g(f(x)) =$ _____, $f \circ g = f(g(x)) =$ _____

EXAMPLE 1

Find the inverse of the function $f(x) = \sqrt{4x - 3}$.

1. Find the Inverse of $g(x) = 2x - 3$.
 - a. $g^{-1}(x) = \frac{x}{2} + 3$
 - b. $g^{-1}(x) = \frac{x+3}{2}$
 - c. $g^{-1}(x) = \frac{x+2}{3}$
 - d. $g^{-1}(x) = \frac{x}{3} + 2$
2. Find the Inverse of $f(x) = \frac{x^3 - 2}{4}$.
 - a. $f^{-1}(x) = 4\left(\frac{x}{2}\right)^3$
 - b. $f^{-1}(x) = \sqrt[3]{4x+2}$
 - c. $f^{-1}(x) = 4\sqrt[3]{x+2}$
 - d. $f^{-1}(x) = 4x^3 + 2$
3. Find the Inverse of $f(x) = \frac{(x+3)}{(x-4)}$, where $x \neq 4$.
 - a. $f^{-1}(x) = \frac{x+4}{x-3}$
 - b. $f^{-1}(x) = \frac{2x-1}{4x-3}$
 - c. $f^{-1}(x) = \frac{4x+3}{x-1}$
 - d. $f^{-1}(x) = \frac{x-1}{2x-1}$
4. Find the Inverse of $f(x) = x^2 + 3$, $x \geq 0$.
 - a. $f^{-1}(x) = \pm\sqrt{x-3}$
 - b. $f^{-1}(x) = 3 - \sqrt{x}$
 - c. $f^{-1}(x) = \sqrt{x-3}$

Homework by 7/20

1. <https://www.khanacademy.org/mission/sat/tasks/6336574552096768>
2. <https://www.khanacademy.org/test-prep/sat/sat-math-practice/new-sat-passport-advanced-mathematics/v/sat-math-p13-harder>
3. <https://www.khanacademy.org/mission/sat/tasks/6516264589934592>
4. <https://www.khanacademy.org/mission/sat/tasks/5966533472108544>
5. <https://www.khanacademy.org/mission/sat/tasks/5815288748916736>
6. <https://www.khanacademy.org/mission/sat/tasks/4774072443813888>
7. <https://www.khanacademy.org/mission/sat/tasks/6549854776279040>

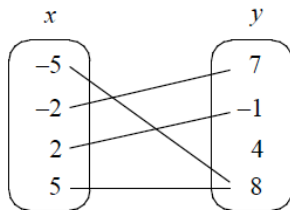
Homework by 7/20

1

What is the domain of the function that contains points at $(-5, 2)$, $(-2, 1)$, $(0, 2)$, and $(4, -3)$?

- A) $\{-3, 1, 2\}$
- B) $\{-2, 1, 0\}$
- C) $\{-5, -2, 1, 2\}$
- D) $\{-5, -2, 0, 4\}$

2



Which of the following relation is a correct representation of the mapping shown above?

- A) $\{(-5, 7), (-2, -1), (2, 4), (5, 8)\}$
- B) $\{(-5, 8), (-2, 7), (2, -1), (5, 8)\}$
- C) $\{(7, -5), (-1, -2), (4, 2), (8, 5)\}$
- D) $\{(8, -5), (7, -2), (-1, 2), (8, 5)\}$

3

If point $(7, b)$ is in Quadrant I and point $(a, -3)$ is in Quadrant III, in which Quadrant is the point (a, b) ?

- A) Quadrant I
- B) Quadrant II
- C) Quadrant III
- D) Quadrant IV

4

If $f(x) = -2x + 7$, what is $f\left(\frac{1}{2}x + 3\right)$ equal to?

- A) $-x + 1$
- B) $-x + 3$
- C) $-x + 5$
- D) $-x + 10$

5

$$g(x) = kx^3 + 3$$

For the function g defined above, k is a constant and $g(-1) = 5$. What is the value of $g(1)$?

- A) -3
- B) -1
- C) 1
- D) 3

6

If $f(x+1) = -\frac{1}{2}x + 6$, what is the value of $f(-3)$?

7

$$f(x) = x^2 - b$$

In the function above, b is a constant. If $f(-2) = 7$, what is the value of $f(b)$?



1

x	-4	0	6
$f(x)$	-4	-1	k

In the table above, if $f(x)$ is a linear function, what is the value of k ?

- A) 2.5
- B) 3
- C) 3.5
- D) 4

2

The graph of a line in the xy -plane has slope $\frac{1}{3}$ and contains the point $(9, 1)$. The graph of a second line passes through the points $(-2, 4)$ and $(5, -3)$. If the two lines intersect at (a, b) , what is the value of $a + b$?

- A) -2
- B) 2
- C) 4
- D) 6

3

Which of the following expressions is equal to 0 for some value of x ?

- A) $5 + |x + 5|$
- B) $5 + |x - 5|$
- C) $-5 + |x + 5|$
- D) $-5 - |x - 5|$

4

Line ℓ in the xy -plane contains points from each of the Quadrants I, III, and IV, but no points from Quadrant II. Which of the following must be true?

- A) The slope of line ℓ is zero.
- B) The slope of line ℓ is undefined.
- C) The slope of line ℓ is positive.
- D) The slope of line ℓ is negative.

5

x	-3	-1	1	5
$f(x)$	9	5	1	-7

The table above shows some values of the linear function f . Which of the following defines f ?

- A) $f(x) = 2x - 3$
- B) $f(x) = -2x + 3$
- C) $f(x) = 2x - 1$
- D) $f(x) = -2x + 1$

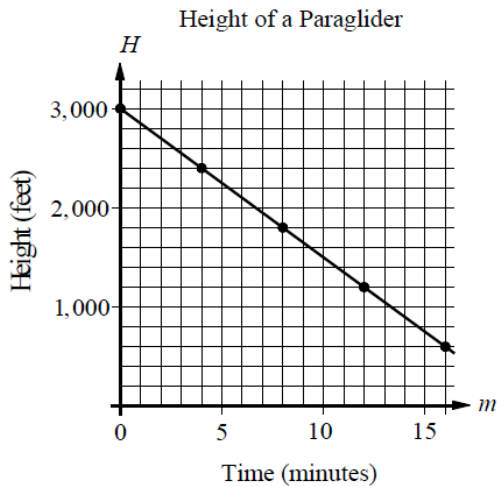
6

If $f(x) = -6x + 1$, what is $f\left(\frac{1}{2}x - 1\right)$ equal to?

- A) $-3x + 7$
- B) $-3x - 5$
- C) $-3x + 1$
- D) $-3x - 1$



Questions 7 and 8 refer to the following information.



The graph above shows the relationship between the height of paraglider H , in feet, and time m , in minutes.

7

Which of the following represents the relationship between H and m ?

- A) $H = -100m + 3000$
- B) $H = -150m + 3000$
- C) $H = -175m + 3000$
- D) $H = -225m + 3000$

8

If the height of the paraglider is 1,350 feet, which of the following best approximates the time the paraglider has been flying?

- A) 10 minutes
- B) 10 minutes and 30 seconds
- C) 11 minutes
- D) 11 minutes and 30 seconds

9

A line in the xy -plane passes through the point $(1, -2)$ and has a slope of $\frac{1}{3}$. Which of the following points lies on the line?

- A) $(3, -2)$
- B) $(2, -\frac{4}{3})$
- C) $(0, -2)$
- D) $(-1, -\frac{8}{3})$

10

$$f(x) = ax + 2$$

In the function above, a is a constant. If

$$f(-1) = 4, \text{ what is the value of } f(-\frac{1}{2})?$$

11

If the slope of the line in the xy -plane that passes through the points $(2, -4)$ and $(6, k)$ is $\frac{3}{2}$, what is the value of k ?

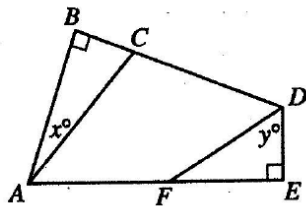
12

$$\frac{1}{3}x - \frac{3}{4}y = -11$$

$$\frac{1}{2}x + \frac{1}{6}y = -1$$

If (x, y) is the solution to the system of equations above, what is the value of $x + y$?





Note: Figure not drawn to scale.

15. In quadrilateral $ABDE$ above, C lies on side BD and F lies on side AE . If AC bisects $\angle BAF$ and DF bisects $\angle BDE$, what is the value of $x + y$?

- (A) 30
- (B) 45
- (C) 60
- (D) 75
- (E) 90

$$ab + cf + gh$$

16. The expression above represents an even integer. At most, how many of the integers a , b , c , f , g , and h could be odd?

- (A) One
- (B) Three
- (C) Four
- (D) Five
- (E) Six



Note: Figure not drawn to scale.

19. If $AD = BE$ in the figure above, then which of the following must be true?

- (A) $AB = EF$
- (B) $AC = CE$
- (C) $AB < DF$
- (D) $AC < CF$
- (E) $BC < CE$

17. If a certain candy company produces x pounds of gumdrops for every y pounds of jelly beans, how many pounds of gumdrops will be produced for n pounds of jelly beans, in terms of x , y , and n ?

- (A) $\frac{nx}{y}$
- (B) $\frac{ny}{x}$
- (C) $\frac{xy}{n}$
- (D) nxy
- (E) $n + x + y$

18. For all values of r , let $\star r$ be defined as

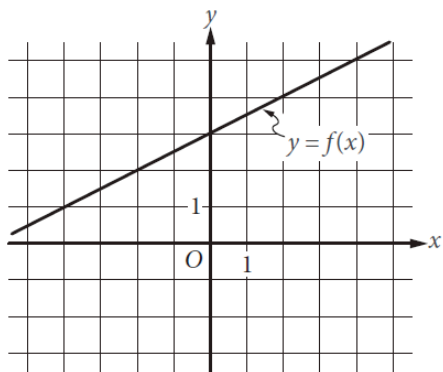
$$\star r = \frac{r + 2}{2}. \text{ If } \star 4 = x, \text{ then } \star x =$$

- (A) $\frac{3}{2}$
- (B) 2
- (C) $\frac{5}{2}$
- (D) 3
- (E) 4

20. If $3^k \cdot 4^m = 12^{20}$ for positive integers k and m , what is the value of $k + m$?

- (A) 7
- (B) 10
- (C) 12
- (D) 20
- (E) 40





The graph of the linear function f is shown in the xy -plane above. The slope of the graph of the linear function g is 4 times the slope of the graph of f . If the graph of g passes through the point $(0, -4)$, what is the value of $g(9)$?

- A) 5
- B) 9
- C) 14
- D) 18

Horsepower and watts are units of measure of power. They are directly proportional such that 5 horsepower is equal to 3730 watts. How much power, in watts, is equal to 2 horsepower?

$$x^2 + 20x + y^2 + 16y = -20$$

The equation above defines a circle in the xy -plane. What are the coordinates of the center of the circle?

- A) $(-20, -16)$
- B) $(-10, -8)$
- C) $(10, 8)$
- D) $(20, 16)$

$$y = x^2 - a$$

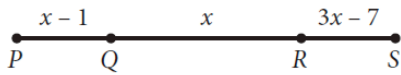
In the equation above, a is a positive constant and the graph of the equation in the xy -plane is a parabola. Which of the following is an equivalent form of the equation?

- A) $y = (x + a)(x - a)$
- B) $y = (x + \sqrt{a})(x - \sqrt{a})$
- C) $y = \left(x + \frac{a}{2}\right)\left(x - \frac{a}{2}\right)$
- D) $y = (x + a)^2$

The painting *The Starry Night* by Vincent van Gogh is rectangular in shape with height 29 inches and width 36.25 inches. If a reproduction was made where each dimension is $\frac{1}{3}$ the corresponding original dimension, what is the height of the reproduction, in inches?



33



Note: Figure not drawn to scale.

On \overline{PS} above, $PQ = RS$. What is the length of \overline{PS} ?

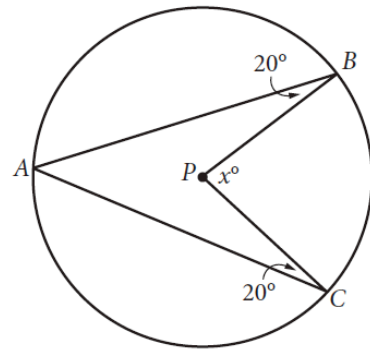
35

A landscaper is designing a rectangular garden. The length of the garden is to be 5 feet longer than the width. If the area of the garden will be 104 square feet, what will be the length, in feet, of the garden?

34

In the xy -plane, the point $(2, 5)$ lies on the graph of the function f . If $f(x) = k - x^2$, where k is a constant, what is the value of k ?

36



Point P is the center of the circle in the figure above. What is the value of x ?

Questions 37 and 38 refer to the following information.

Ms. Simon's Workday Morning Drive

Segment of drive	Distance (miles)	Average driving speed with no traffic delay (mph)
From home to freeway entrance	0.6	25
From freeway entrance to freeway exit	15.4	50
From freeway exit to workplace	1.4	35

Ms. Simon drives her car from her home to her workplace every workday morning. The table above shows the distance, in miles, and her average driving speed, in miles per hour (mph), when there is no traffic delay, for each segment of her drive.

37

One morning, Ms. Simon drove directly from her home to her workplace in 24 minutes. What was her average speed, in miles per hour, during her drive that morning?

38

If Ms. Simon starts her drive at 6:30 a.m., she can drive at her average driving speed with no traffic delay for each segment of the drive. If she starts her drive at 7:00 a.m., the travel time from the freeway entrance to the freeway exit increases by 33% due to slower traffic, but the travel time for each of the other two segments of her drive does not change. Based on the table, how many more minutes does Ms. Simon take to arrive at her workplace if she starts her drive at 7:00 a.m. than if she starts her drive at 6:30 a.m.? (Round your answer to the nearest minute.)

Math Test
Calculator Answers

- | | | | |
|------|------|------|----------------------------------|
| 1 D | 11 A | 21 A | 31 1492 |
| 2 C | 12 D | 22 C | 32 $29\frac{2}{3}$, 9.66, 9.67 |
| 3 A | 13 D | 23 D | 33 7 |
| 4 B | 14 A | 24 B | 34 9 |
| 5 C | 15 A | 25 D | 35 13 |
| 6 B | 16 D | 26 B | 36 80 |
| 7 A | 17 D | 27 C | 37 43, 43.5, 44, $87\frac{1}{2}$ |
| 8 C | 18 C | 28 C | 38 6 |
| 9 B | 19 B | 29 B | |
| 10 A | 20 D | 30 B | |

Day 14. 7/21 Wednesday

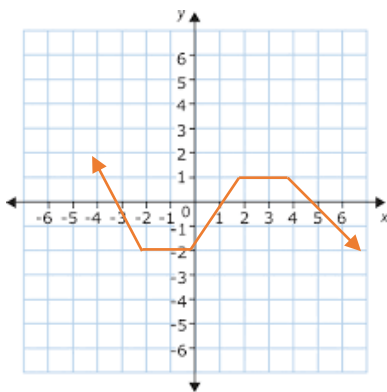
Graph Transformations Exponential Functions

Finish pages 24-36 by 7/22

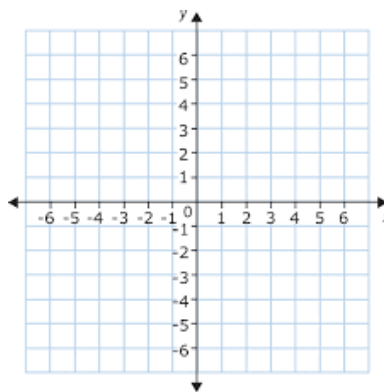
Graph Transformations

<i>function</i>	<i>transformation</i>	<i>point</i>
$f(x)$		(a, b)
$-f(x)$		
$c \cdot f(x)$		
$f(x) + d$		
$f(x + c)$		
$f(-x)$		
$f(c \cdot x)$		

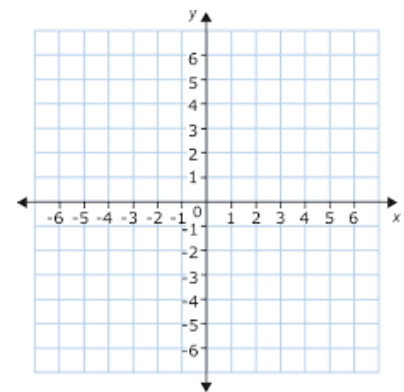
$f(x)$



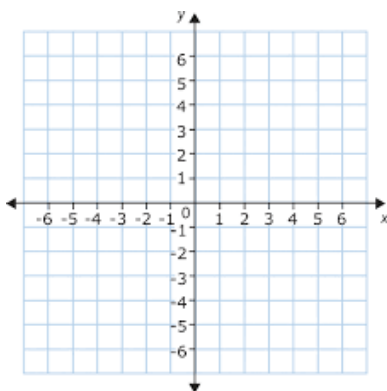
$-f(x)$



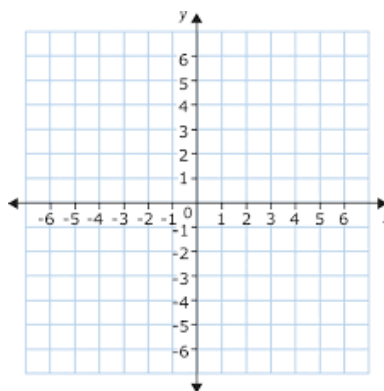
$3f(x)$



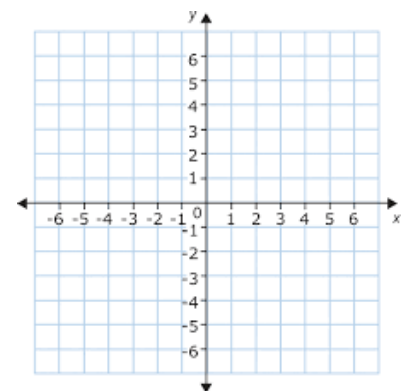
$f(x) + 4$



$f(x - 3)$



$f(-x)$



Graph transformations - Classwork

EXERCISE  Match.

1. x^2

4. $x^2 + 1$

7. $(x + 1)^2$

2. $(x + 1)^2 + 1$

5. $(x + 1)^2 - 1$

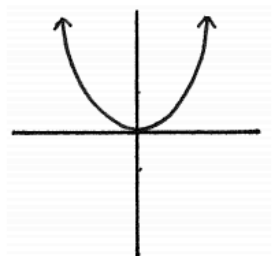
8. $(x - 1)^2 + 1$

3. $(x - 1)^2 - 1$

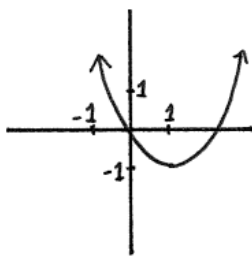
6. $-(x + 1)^2$

9. $-(x - 1)^2$

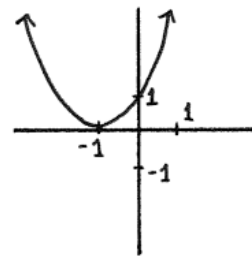
A.)



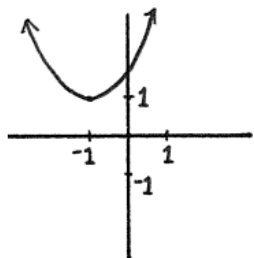
B.)



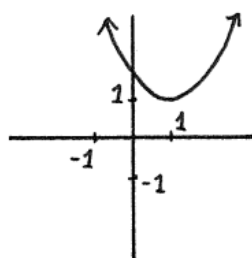
C.)



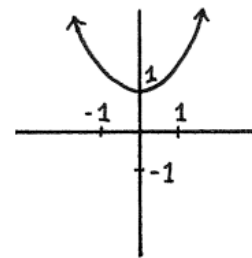
D.)



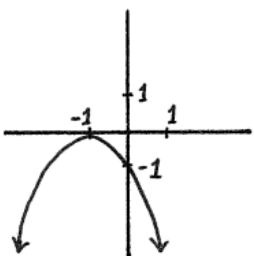
E.)



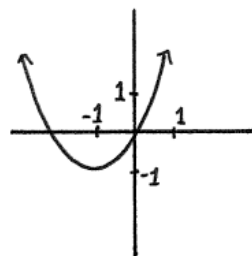
F.)



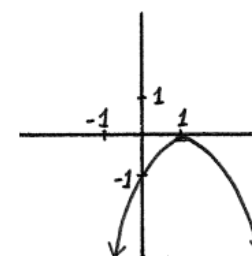
G.)



H.)

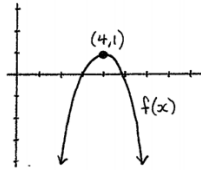


I.)



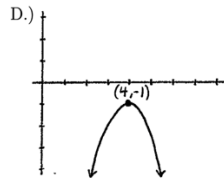
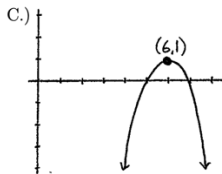
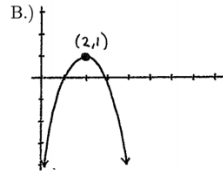
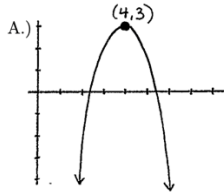
Graph Transformations - Homework

PROBLEM 1

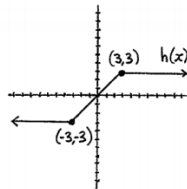


Given the graph of $f(x)$ above, match the following four functions with their graphs.

- 13.) $f(x)+2$ 14.) $f(x)-2$ 15.) $f(x+2)$ 16.) $f(x-2)$

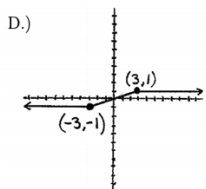
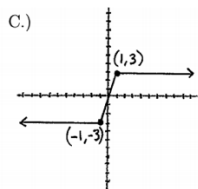
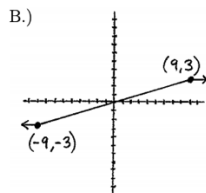
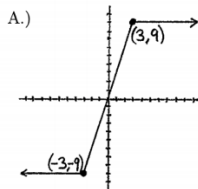


PROBLEM 3

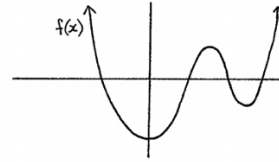


Given the graph of $h(x)$ above, match the following four functions with their graphs.

- 29.) $3h(x)$ 30.) $\frac{1}{3}h(x)$ 31.) $h(3x)$ 32.) $h(\frac{x}{3})$

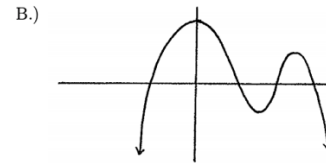
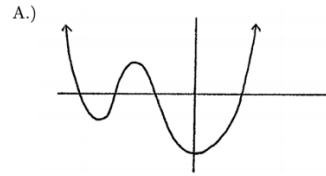


PROBLEM 2



Given the graph of $f(x)$ above, match the following two functions with their graphs.

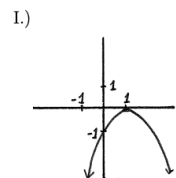
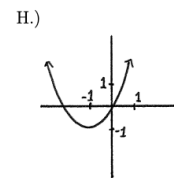
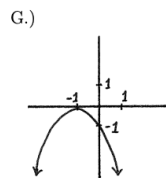
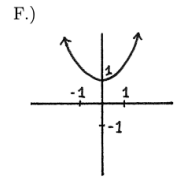
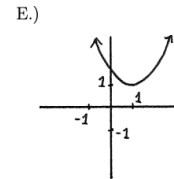
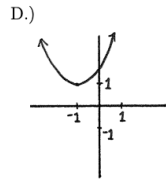
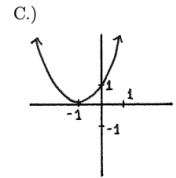
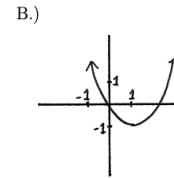
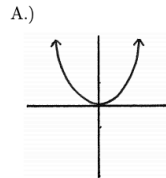
- 23.) $-f(x)$ 24.) $f(-x)$



PROBLEM 4

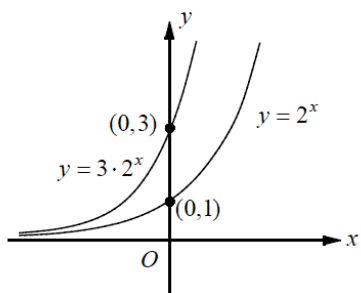
For #33-41, match the numbered functions with their lettered graphs.

- 33.) x^2 34.) $x^2 + 1$ 35.) $(x + 1)^2$
 36.) $(x + 1)^2 + 1$ 37.) $(x + 1)^2 - 1$ 38.) $(x - 1)^2 + 1$
 39.) $(x - 1)^2 - 1$ 40.) $-(x + 1)^2$ 41.) $-(x - 1)^2$

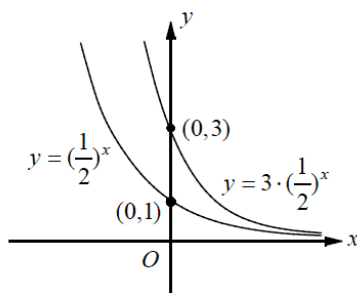


Exponential Functions - In Class Notes

An **exponential function** is a function of the form $f(x) = ab^x$, in which $a \neq 0$, $b > 0$, and $b \neq 1$.



If $b > 1$, the graph rises as x increases. The graph shows exponential growth.

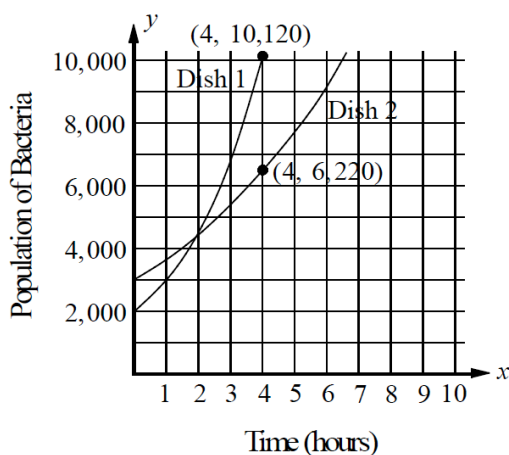


If $0 < b < 1$, the graph falls as x increases. The graph shows exponential decay.

Any function of the form $f(x) = ab^x$, in which $a > 0$, $b > 0$, and $b \neq 1$, the domain is the set of all real numbers and the range is the set of positive real numbers.

EXAMPLE 1

In the diagram below, each exponential curve represents the population of bacteria in a petri dish as a function of time, in hours. At time $t = 0$, the population of Dish 1 is 2,000 and the population of Dish 2 is 3,000.

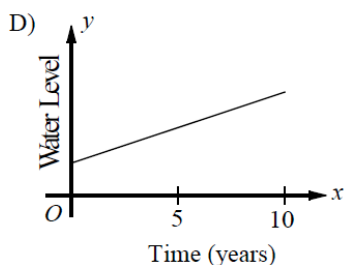
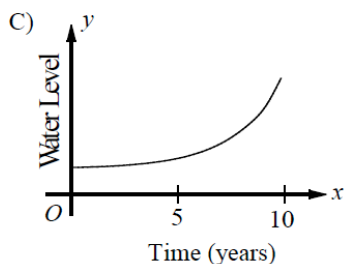
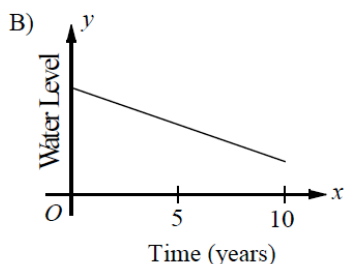
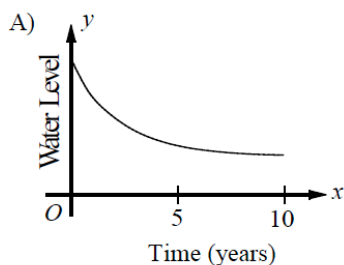


- At time $t = 0$, the number of bacteria in Dish 2 is what percent more than the number of bacteria in Dish 1?
- Find the average growth rate of bacteria in Dish 1 and in Dish 2 from time $t = 0$ to time $t = 4$.

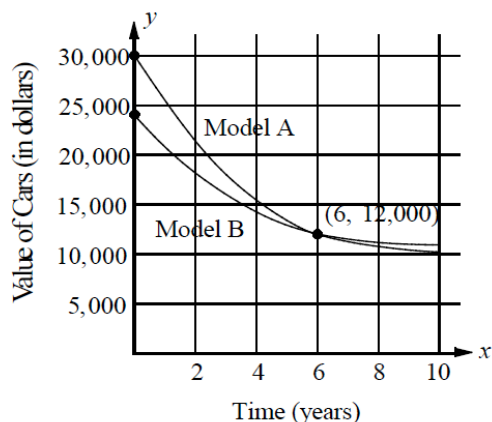
Exponential functions - Homework

1

During a decade of continuous drought, the water level of a lake has decreased by 10 percent each year. Which of the following graphs could model the water level of the lake as a function of time?



2



In the graph above, each exponential curve represents the values, in dollars, of two different cars as a function of time in years. At time $t = 0$, the price of model A was \$30,000 and the price of model B was \$24,000. At time $t = 6$, the price of both models were \$12,000.

Based on the graphs above, which of the following must be true?

- I. At time $t = 0$, the price of model A was 25% more than the price of model B .
- II. At time $t = 0$, the price of model B was 20% less than the price of model A .
- III. From time $t = 0$ to $t = 6$, the average rate of decrease in the value of model A was 1.5 times the average rate of decrease in the value of model B .

- A) I and II only
- B) I and III only
- C) II and III only
- D) I, II, and III

3

If $f(x) = 12,000(0.9)^x$ and $g(x) = 14,000(0.85)^x$, what is the value of $g(2) - f(2)$?

Exponential growth & decay – In-class notes

Compound Interest Formulas

If initial amount P is invested at annual interest rate r , the investment will grow to final amount A in t years. $A = P(1+r)^t$

We can use the same formula for the population or value of goods that is increasing or decreasing.

Exponential Growth and Doubling-Time Growth Formula

If a population is increasing at a constant rate r each year, the population at the end of t years would be $A = P(1+r)^t$.


If an initial population of size P doubles every d years (or any other unit of time), the final number A in t years is given by $A = P(2)^{t/d}$.


Exponential Decay and Half-Life Decay Formula


If a population is decreasing at a constant rate r each year, the population at the end of t years would be $A = P(1-r)^t$.

The **half-life** of a substance is the amount of time it takes for half of the substance to decay.


If an initial population of size P has a half-life of d years (or any other unit of time), the final number A in t years is given by $A = P\left(\frac{1}{2}\right)^{t/d}$.


EXAMPLE 1  The population of a western town doubles in size every year. If the population of town is 8,000, what will the population be 18 years from now?


EXAMPLE 2  The population of a western town doubles in size every 2 years. If the population of town is 8,000, what will the population be 18 years from now?


EXAMPLE 3  The population of a western town doubles in size every 6 months. If the population of town is 8,000, what will the population be 18 years from now?

Write the equation for each question, then solve.

EXAMPLE 4  Mark invests \$1500 at a rate of 6% interest compounded annually. How much is the investment worth in 5 years?

EXAMPLE 5  The price of a new automobile is \$28,000. If the value of the automobile decreases 12% per year, what will be the price of the automobile after 5 years?

EXAMPLE 6  The half-life of carbon-14 is 6000 years. How much of 800g of this substance will remain after 30,000 years?

EXAMPLE 7  12 grams (g) of a chemical are added to a metal. The amount A , in grams, of the chemical remaining during a reaction with a metal plate decreases by 0.5 g per second. If instead the plate were dissolved, the amount A' , in grams, of chemical remaining would decrease by half of itself every 4 seconds. How many grams greater is A than A' after 12 seconds?

Answer:

EXAMPLE 8

An old computer program which computes the n^{th} Fibonacci number takes 0.05 microseconds (μs) to compute the 1^{st} number. After this, each number takes twice as long as the previous number to compute. A computer engineer designs a new program which takes $0.8 \mu s$ to compute the 1^{st} Fibonacci number. After this, each number takes $0.6 \mu s$ longer than the previous number to compute. How much longer in microseconds does it take the old program to compute the 8^{th} Fibonacci number compared to the new program?

- (A) 0.8 ...
- (B) 1.4 ...
- (C) 5.0 ...
- (D) 6.4 ...

EXAMPLE 9

$$r = 35.7 + 1.37t$$

The equation above relates the urbanization rate r , as a percent, in a particular country to the number of years t since 2000. Which of the following statements best describes the relationship between the years since 2000 and the urbanization rate?

- (A) It is linear because the urbanization rate increases by 35.7 each year. ...
- (B) It is linear because the urbanization rate increases by 1.37 each year. ...
- (C) It is exponential because the urbanization rate increases by 37% each year. ...
- (D) It is exponential because the urbanization rate increases by a factor of 35.7 each year. ...

Exponential growth & decay – Homework

1

The number of rabbits in a certain population doubles every 40 days. If the population starts with 12 rabbits, which of the following gives the total number of rabbits in the population after t days?

- A) $12(2)(\frac{t}{40})$
- B) $12(2)(\frac{40}{t})$
- C) $12(2)^{\frac{40}{t}}$
- D) $12(2)^{\frac{t}{40}}$

2

Population P of a town is 80,000 this year. If the population of the town decreases at a rate of 4 percent each year, which of the following expressions gives population P after t years?

- A) $80,000(0.6)^t$
- B) $80,000(0.96)^t$
- C) $80,000(0.96t)$
- D) $80,000(1 - 0.04t)$

3

A house bought ten years ago for \$150,000 was sold for \$240,000 this year. Which of the following equations can be used to solve the annual growth rate r of the value of the house?

- A) $240,000 = 150,000(1 + \frac{r}{10})$
- B) $240,000 = 150,000(1 + 10r)$
- C) $240,000 = 150,000(1 + r)^{10}$
- D) $240,000 = 150,000(r)^{10}$

4

A certain radioactive substance has a half-life of 12 days. This means that every 12 days, half of the original amount of the substance decays. If there are 128 milligrams of the radioactive substance today, how many milligrams will be left after 48 days?

- A) 4
- B) 8
- C) 16
- D) 32

Questions 5 and 6 refer to the following information.

Evelyn deposited \$3,000 into her bank account, which earns 4 percent interest compounded annually. She uses the expression $\$3,000(x)^t$ to find the value of the account after t years.

5

What is the value of x in the expression?

6

Evelyn deposited the same amount into an account that earns 5 percent interest rate compounded annually. How much more money than her original deposit in the account with 4 percent interest rate compounded annually will she have earned in 10 years?

(Round your answer to the nearest dollar.)



Advanced Math Real Test – Homework

39. If $\frac{q}{r} = 3$ and $\frac{r}{t} = 5$ and $\frac{t}{u} = 7$, what is the value of $\frac{qu}{t^2}$?

14. All the students ranking in the highest one-third of Maria's high school class went to college. If Maria ranked in the higher half of her high school class, which of the following can be inferred?

- I. Maria ranked higher than more than $\frac{3}{10}$ of her class.
- II. Maria ranked in the highest quarter of her class.
- III. Maria did not go to college.

- (A) I only
- (B) II only
- (C) III only
- (D) II and III only
- (E) I, II, and III

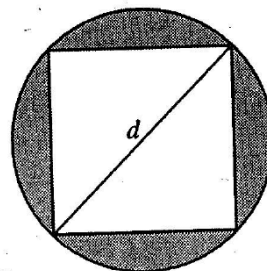
15. For all positive integers x and y , if $\#$ is defined by $x \# y = (x + y) - 2$, then $(3 \# 4) \# 5 =$

- (A) 4
- (B) 6
- (C) 8
- (D) 10
- (E) 58

40. The surface area of a cube is 54 square inches. What is the volume of the cube, in cubic inches?

16. If a and b are integers greater than 100 such that $a + b = 300$, which of the following could be the exact ratio of a to b ?

- (A) 9 to 1
- (B) 5 to 2
- (C) 5 to 3
- (D) 4 to 1
- (E) 3 to 2



17. In the figure above, a square is inscribed in a circle with diameter d . What is the sum of the areas of the shaded regions, in terms of d ?

- (A) $d^2 \left(\frac{\pi}{4} - \frac{1}{2} \right)$
- (B) $d^2 \left(\frac{\pi}{4} - \frac{1}{4} \right)$
- (C) $d^2 \left(\frac{\pi}{2} - \frac{1}{2} \right)$
- (D) $d^2 (\pi - 2)$
- (E) $d^2 (\pi - 1)$

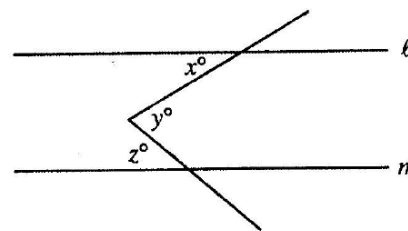


18. For how many values of x is
 $x(x - 1)(x - 2) - (x - 1)(x - 2)(x - 3) = 0$?

(A) One
(B) Two
(C) Three
(D) Four
(E) More than four

-
19. If m and x are positive integers and $5^{(x+1)} = 5m$,
which of the following equals 5^x ?

(A) $m^{(x+1)}$
(B) m
(C) $5^{(m-1)}$
(D) $m - 1$
(E) $\frac{m}{5}$

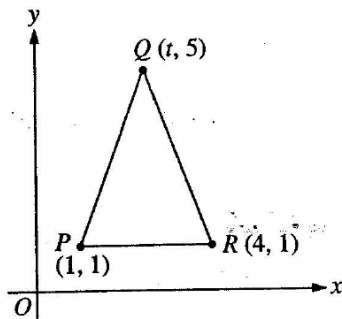


20. In the figure above, lines l and m are parallel. In terms of x and y , which of the following is equal to z ?

(A) $y - x$
(B) $90 + x - y$
(C) $180 - y - x$
(D) $180 + y - x$
(E) $y - x - 180$



35. A television set is on sale for 20 percent off the regular price of \$500. If Ms. Jones purchases the set on sale and pays sales tax of 5 percent of the purchase price, what is the exact dollar amount that she pays for the television set? (Disregard the \$ sign when gridding your answer.)



36. In the figure above, $\angle P$ and $\angle R$ have the same measure. What is the value of t ?

39. The sum of r and p is equal to twice s , and p is 36 less than twice the sum of r and s . What is the value of r ?

$$\begin{array}{r} 2R2 \\ \times T \\ \hline 1P74 \end{array}$$

37. In the correctly worked multiplication problem above, P , R , and T represent different digits. What is the value of P ?

38. If $\frac{ax}{cy} = \frac{3}{4}$ and $\frac{a}{c} = \frac{1}{2}$, what is the value of $\frac{y}{x}$?

40. In a right triangle, 7 times the degree measure of the smallest angle is equal to 5 times the degree measure of the next-smaller angle. What is the degree measure of the smallest angle?

Day 15. 7/23 Friday page 37-46

14

Type of surgeon	Major professional activity		Total
	Teaching	Research	
General	258	156	414
Orthopedic	119	74	193
Total	377	230	607

In a survey, 607 general surgeons and orthopedic surgeons indicated their major professional activity. The results are summarized in the table above. If one of the surgeons is selected at random, which of the following is closest to the probability that the selected surgeon is an orthopedic surgeon whose indicated professional activity is research?

- A) 0.122
- B) 0.196
- C) 0.318
- D) 0.379

15

A polling agency recently surveyed 1,000 adults who were selected at random from a large city and asked each of the adults, "Are you satisfied with the quality of air in the city?" Of those surveyed, 78 percent responded that they were satisfied with the quality of air in the city. Based on the results of the survey, which of the following statements must be true?

- I. Of all adults in the city, 78 percent are satisfied with the quality of air in the city.
- II. If another 1,000 adults selected at random from the city were surveyed, 78 percent of them would report they are satisfied with the quality of air in the city.
- III. If 1,000 adults selected at random from a different city were surveyed, 78 percent of them would report they are satisfied with the quality of air in the city.

- A) None
- B) II only
- C) I and II only
- D) I and III only



Questions 16-18 refer to the following information.

Species of tree	Growth factor
Red maple	4.5
River birch	3.5
Cottonwood	2.0
Black walnut	4.5
White birch	5.0
American elm	4.0
Pin oak	3.0
Shagbark hickory	7.5

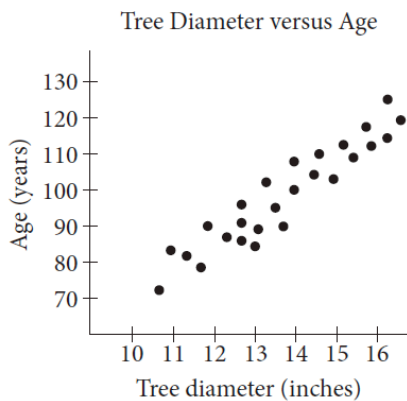
One method of calculating the approximate age, in years, of a tree of a particular species is to multiply the diameter of the tree, in inches, by a constant called the growth factor for that species. The table above gives the growth factors for eight species of trees.

16

According to the information in the table, what is the approximate age of an American elm tree with a diameter of 12 inches?

- A) 24 years
- B) 36 years
- C) 40 years
- D) 48 years

17



The scatterplot above gives the tree diameter plotted against age for 26 trees of a single species. The growth factor of this species is closest to that of which of the following species of tree?

- A) Red maple
- B) Cottonwood
- C) White birch
- D) Shagbark hickory

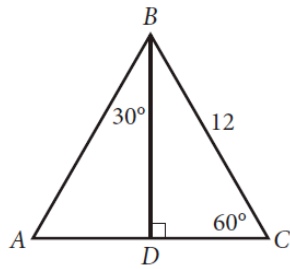
18

If a white birch tree and a pin oak tree each now have a diameter of 1 foot, which of the following will be closest to the difference, in inches, of their diameters 10 years from now? (1 foot = 12 inches)

- A) 1.0
- B) 1.2
- C) 1.3
- D) 1.4



19



In $\triangle ABC$ above, what is the length of \overline{AD} ?

- A) 4
- B) 6
- C) $6\sqrt{2}$
- D) $6\sqrt{3}$

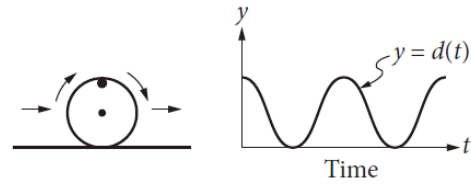
21

$$\frac{a-b}{a} = c$$

In the equation above, if a is negative and b is positive, which of the following must be true?

- A) $c > 1$
- B) $c = 1$
- C) $c = -1$
- D) $c < -1$

20



The figure on the left above shows a wheel with a mark on its rim. The wheel is rolling on the ground at a constant rate along a level straight path from a starting point to an ending point. The graph of $y = d(t)$ on the right could represent which of the following as a function of time from when the wheel began to roll?

- A) The speed at which the wheel is rolling
- B) The distance of the wheel from its starting point
- C) The distance of the mark on the rim from the center of the wheel
- D) The distance of the mark on the rim from the ground

22

In State X, Mr. Camp's eighth-grade class consisting of 26 students was surveyed and 34.6 percent of the students reported that they had at least two siblings. The average eighth-grade class size in the state is 26. If the students in Mr. Camp's class are representative of students in the state's eighth-grade classes and there are 1,800 eighth-grade classes in the state, which of the following best estimates the number of eighth-grade students in the state who have fewer than two siblings?

- A) 16,200
- B) 23,400
- C) 30,600
- D) 46,800



Questions 23 and 24 refer to the following information.

Townsend Realty Group Investments		
Property address	Purchase price (dollars)	Monthly rental price (dollars)
Clearwater Lane	128,000	950
Driftwood Drive	176,000	1,310
Edgemont Street	70,000	515
Glenview Street	140,000	1,040
Hamilton Circle	450,000	3,365

The Townsend Realty Group invested in the five different properties listed in the table above. The table shows the amount, in dollars, the company paid for each property and the corresponding monthly rental price, in dollars, the company charges for the property at each of the five locations.

23

The relationship between the monthly rental price r , in dollars, and the property's purchase price p , in thousands of dollars, can be represented by a linear function. Which of the following functions represents the relationship?

- A) $r(p) = 2.5p - 870$
- B) $r(p) = 5p + 165$
- C) $r(p) = 6.5p + 440$
- D) $r(p) = 7.5p - 10$

24

Townsend Realty purchased the Glenview Street property and received a 40% discount off the original price along with an additional 20% off the discounted price for purchasing the property in cash. Which of the following best approximates the original price, in dollars, of the Glenview Street property?

- A) \$350,000
- B) \$291,700
- C) \$233,300
- D) \$175,000

25

A psychologist set up an experiment to study the tendency of a person to select the first item when presented with a series of items. In the experiment, 300 people were presented with a set of five pictures arranged in random order. Each person was asked to choose the most appealing picture. Of the first 150 participants, 36 chose the first picture in the set. Among the remaining 150 participants, p people chose the first picture in the set. If more than 20% of all participants chose the first picture in the set, which of the following inequalities best describes the possible values of p ?

- A) $p > 0.20(300 - 36)$, where $p \leq 150$
- B) $p > 0.20(300 + 36)$, where $p \leq 150$
- C) $p - 36 > 0.20(300)$, where $p \leq 150$
- D) $p + 36 > 0.20(300)$, where $p \leq 150$

26

The surface area of a cube is $6\left(\frac{a}{4}\right)^2$, where a is a positive constant. Which of the following gives the perimeter of one face of the cube?

- A) $\frac{a}{4}$
- B) a
- C) $4a$
- D) $6a$

27

The mean score of 8 players in a basketball game was 14.5 points. If the highest individual score is removed, the mean score of the remaining 7 players becomes 12 points. What was the highest score?

- A) 20
- B) 24
- C) 32
- D) 36

Math Test Calculator Answers

1 D	11 A	21 A	31 1492
2 C	12 D	22 C	32 $29/3$, 9.66, 9.67
3 A	13 D	23 D	33 7
4 B	14 A	24 B	34 9
5 C	15 A	25 D	35 13
6 B	16 D	26 B	36 80
7 A	17 D	27 C	37 43, 43.5, 44, $87/2$
8 C	18 C	28 C	38 6
9 B	19 B	29 B	
10 A	20 D	30 B	



8

In air, the speed of sound S , in meters per second, is a linear function of the air temperature T , in degrees Celsius, and is given by $S(T) = 0.6T + 331.4$. Which of the following statements is the best interpretation of the number 331.4 in this context?

- A) The speed of sound, in meters per second, at 0°C
- B) The speed of sound, in meters per second, at 0.6°C
- C) The increase in the speed of sound, in meters per second, that corresponds to an increase of 1°C
- D) The increase in the speed of sound, in meters per second, that corresponds to an increase of 0.6°C

9

$$y = x^2$$
$$2y + 6 = 2(x + 3)$$

If (x, y) is a solution of the system of equations above and $x > 0$, what is the value of xy ?

- A) 1
- B) 2
- C) 3
- D) 9

10

If $a^2 + b^2 = z$ and $ab = y$, which of the following is equivalent to $4z + 8y$?

- A) $(a + 2b)^2$
- B) $(2a + 2b)^2$
- C) $(4a + 4b)^2$
- D) $(4a + 8b)^2$



11

The volume of right circular cylinder A is 22 cubic centimeters. What is the volume, in cubic centimeters, of a right circular cylinder with twice the radius and half the height of cylinder A?

- A) 11
- B) 22
- C) 44
- D) 66

12

Which of the following is equivalent to $9^{\frac{3}{4}}$?

- A) $\sqrt[3]{9}$
- B) $\sqrt[4]{9}$
- C) $\sqrt{3}$
- D) $3\sqrt{3}$

13

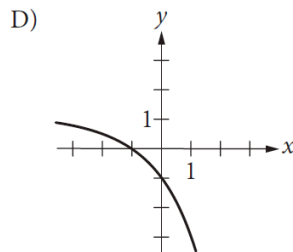
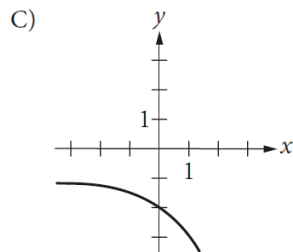
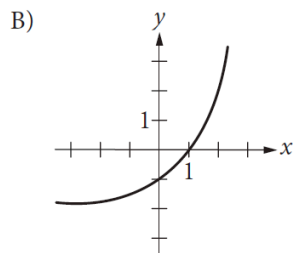
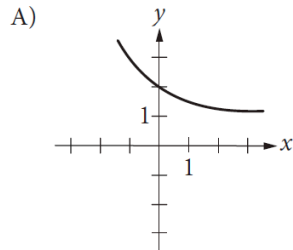
At a restaurant, n cups of tea are made by adding t tea bags to hot water. If $t = n + 2$, how many additional tea bags are needed to make each additional cup of tea?

- A) None
- B) One
- C) Two
- D) Three



$$f(x) = 2^x + 1$$

The function f is defined by the equation above. Which of the following is the graph of $y = -f(x)$ in the xy -plane?



Alan drives an average of 100 miles each week. His car can travel an average of 25 miles per gallon of gasoline. Alan would like to reduce his weekly expenditure on gasoline by \$5. Assuming gasoline costs \$4 per gallon, which equation can Alan use to determine how many fewer average miles, m , he should drive each week?

A) $\frac{25}{4}m = 95$

B) $\frac{25}{4}m = 5$

C) $\frac{4}{25}m = 95$

D) $\frac{4}{25}m = 5$



16

Maria plans to rent a boat. The boat rental costs \$60 per hour, and she will also have to pay for a water safety course that costs \$10. Maria wants to spend no more than \$280 for the rental and the course. If the boat rental is available only for a whole number of hours, what is the maximum number of hours for which Maria can rent the boat?

17

$$2(p + 1) + 8(p - 1) = 5p$$

What value of p is the solution of the equation above?

19

$$\frac{2x + 6}{(x + 2)^2} - \frac{2}{x + 2}$$

The expression above is equivalent to $\frac{a}{(x + 2)^2}$,

where a is a positive constant and $x \neq -2$.

What is the value of a ?

18

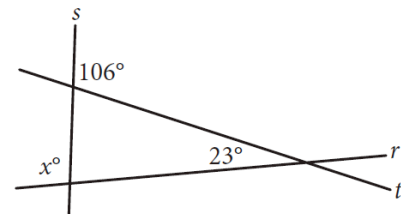
$$\begin{aligned} \frac{1}{2}(2x + y) &= \frac{21}{2} \\ y &= 2x \end{aligned}$$

The system of equations above has solution (x, y) .

What is the value of x ?

20

Intersecting lines r , s , and t are shown below.



What is the value of x ?

Math Test

No Calculator Answers

1 D	11 C
2 A	12 D
3 B	13 B
4 C	14 C
5 D	15 D
6 A	16 4
7 C	17 $6/5$, 1.2
8 A	18 $21/4$, 5.25
9 A	19 2
10 B	20 97

Check your answers with explanations from the website!

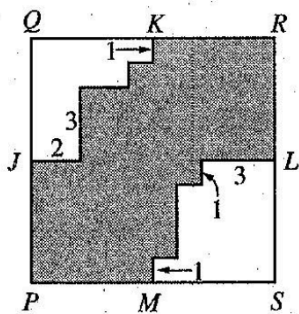


SEM Education

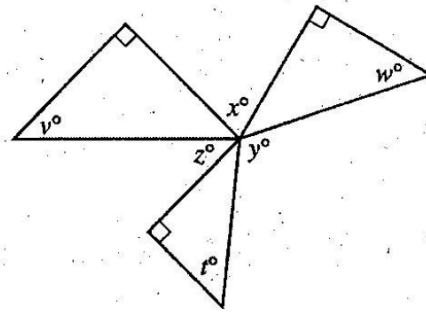
Tutoring | Test Prep | College Prep

35. If the circumference of one circle is 16π and the circumference of another circle is 17π , what is the (positive) difference in length between the radii of these two circles?

36. If $\frac{x+x+x}{3} + \frac{w+w}{2} = 5$, what is the value of $x+w$?



39. The area of square $PQRS$ above is 100. Points J , K , L , and M are midpoints of the sides, and all intersecting line segments are perpendicular. What is the



Note: Figure not drawn to scale.

37. In the figure above, if $t = 45$, $v = 60$, and $w = 60$, what is the value of $x + y + z$?

38. If the sum of the squares of five consecutive integers is 15, what is one possible value of the greatest of the five integers?

RESULTS OF AN ELECTION

Candidate	Number of Votes
X	7,400
Y	2,375
Z	5,250

40. The results for three candidates in an election are shown in the table above. If each voter voted for exactly one candidate, what is the fewest number of voters who would have had to vote differently in order for Candidate Z to have received more votes than Candidate X ?