Day 16. 7/27 Monday

Angles, Triangles
Finish pages 1-22 by 7/28

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Lines

- A line: ⟷
- Segment: ⊶⊸
- Ray: ⊶⟶
- Segment addition postulate: $\overline{AB} + \overline{BC} = \overline{AC}$

  A ⊢———⊢———⊣ C
      B
- Midpoint: M
- Segment bisector: ℓ
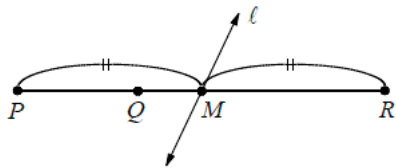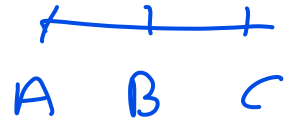
Line ℓ is a segment bisector.

EXAMPLE 1 ✎

Points A, B, M and C lie on the line as shown below. Point M is the midpoint of $\overline{AC}$.
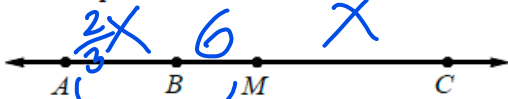
a. Which ray is opposite to ray BC?

b. If $BM = 6$ and $AB = \dfrac{2}{3}MC$, what is the length of AM?

$\overrightarrow{BA}$

$\dfrac{2}{3}x + 6 = x$

$2x + 18 = 3x$

$\boxed{18 = x}$

Angles

- Acute angle: $< 90°$

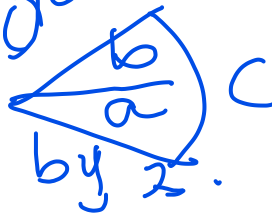- Right angle: $= 90°$

- Obtuse angle: $> 90°$

- Straight angle: $= 180°$

- Angle addition postulate: $\angle a + \angle b = \angle c$

- Angle bisector: $\div$ angle by 2.

- Vertical angles: Vertical angles

- Supplementary angles: $+180°$

- Complementary angles: $90°$

EXAMPLE 1 ✎ Find the number of degrees in an angle which is 42 less than its complement.

$x \qquad 90-x$

$x = (90-x) - 42$

EXAMPLE 2 ✎ Find the number of degrees in an angle which is 120 less than its supplement.

$x \qquad 180-x$

$x = (180-x) - 120$

EXAMPLE 3 ✎ An angle is five times its supplement. Find both angles.

$x \qquad 180-x$

$x = 5(180-x)$

EXAMPLE 4 ✎ The supplement of an angle exceeds the angle by 60 degrees. Find both angles.
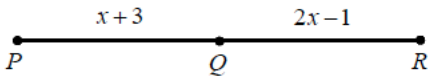
$180-x \qquad x$

$180-x = x + 60$

EXAMPLE 5 ✎ The complement of an angle is 30 less than twice the angle. Find both angles.

$90-x \qquad 90-x = 2x - 30$

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3

Lines Practice: In-Class Work

1

$$x+3 \qquad\qquad 2x-1$$

P   Q   R
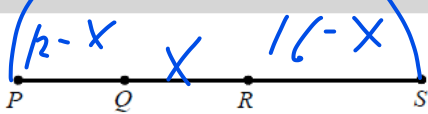
In the figure above, Q is the midpoint of PR. If $PQ = x+3$ and $QR = 2x-1$, what is the length of segment PR?

A) 4

B) 7

C) 11

D) 14

(handwritten) $x+3=2x-1$

$4=x$

(handwritten) $3x$

2

(handwritten) $12-x$ x $16-x$

P   Q   R   S

Note: Figure not drawn to scale.

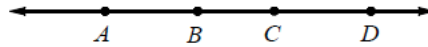On the segment PS above, $PR = 12$, $QS = 16$, and $QR = \dfrac{1}{3}PS$. What is the length of PS?

A) 19

B) 20

C) 21

D) 22

(handwritten) $12+16-x=3x$

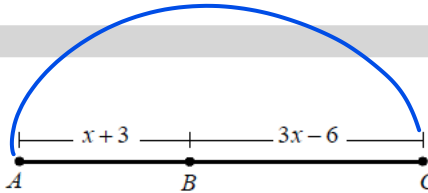$28=4x$

$x=7$

3

A   B   C   D

In the figure above, which of the following are opposite rays?

A) Ray AB and Ray CD

B) Ray CA and Ray CD

C) Ray DA and Ray AD

D) Ray CA and Ray BD

4

(handwritten arc)

├── $x+3$ ──┼── $3x-6$ ──┤

A   B   C

Note: Figure not drawn to scale.

In the figure above, $AB = \dfrac{2}{3}BC$. What is the length of AC?
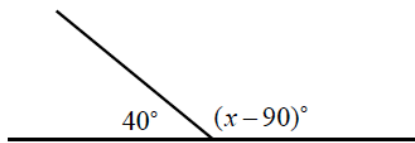
A) 15

B) 18

C) 21

D) 25

(handwritten) $x+3=\dfrac{2}{3}(3x-6)$
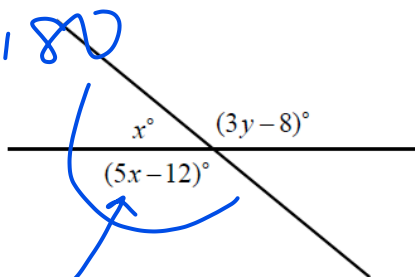
Angles Practice : In-Class Work

1

$40°$ $(x-90)°$

In the figure above, what is the value of x ?

A) 140

B) 160

C) 190

D) 230

(handwritten: $-50 + X = 180$)

2

$x°$ $(3y-8)°$

$(5x-12)°$

Note: Figure not drawn to scale.

In the figure above, what is the values of y ?

A) 52

B) 60

C) 68

D) 76

(handwritten: $6X-12=180$, $X=32$, $148=34-8$)

3

(handwritten: $11x+6$ C $8x-12$ B $4x-6$ D O A)
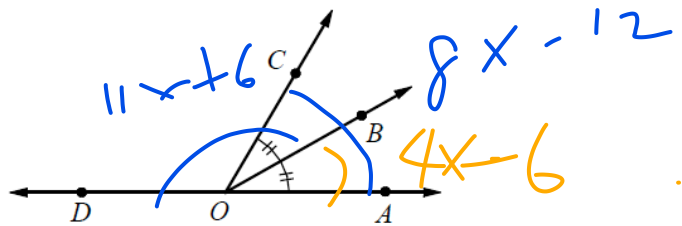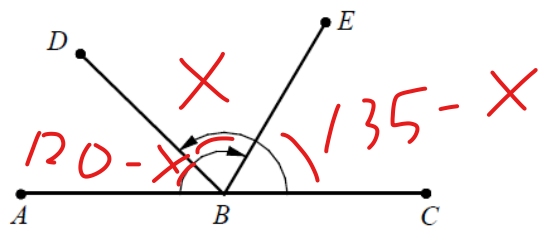
Note: Figure not drawn to scale.

In the figure above, ray OB bisects $\angle COA$. If $m\angle DOB = 11x+6$ and $m\angle COA = 8x-12$, what is the measure of $\angle DOC$?

A) 92

B) 96

C) 102

D) 108

(handwritten: $11X+6+4X-6=180$, $X=12$)

4

D E
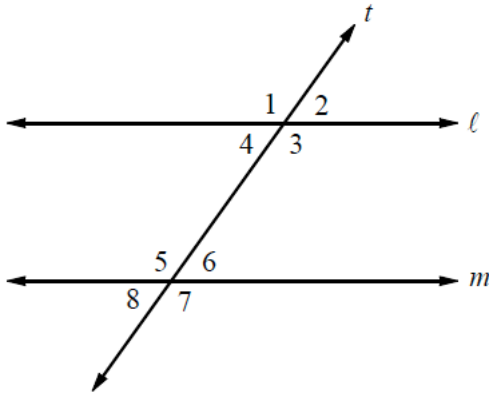
$120-X$ X $135-X$

A B C

Note: Figure not drawn to scale.

In the figure above, $m\angle ABE = 120°$ and $m\angle CBD = 135°$. What is the measure of $\angle DBE$?

A) 63

B) 68

C) 75

D) 79

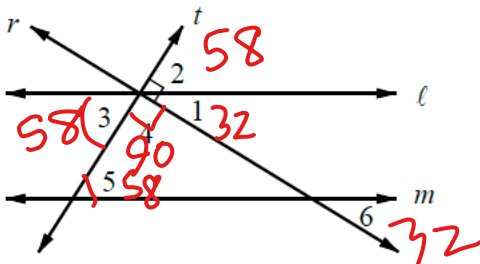*(handwritten: $255-X-X+X=180$, $X=75$)*

Parallel Lines – In class Notes

- Corresponding angles: $\angle 1 \cong \angle 5$
- Alternate interior angles: $\angle 4 \cong \angle 6$
- Alternate exterior angles: $\angle 1 \cong \angle 7$
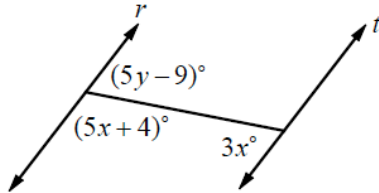- Consecutive interior angles: $\angle 3 + \angle 6 = 180$

EXAMPLE ✏

☐ In the figure below, $\ell \parallel m$, $r \perp t$ and $m\angle 1 = 32$. Lines ℓ, r, and t intersect at one point. Find $m\angle 2$, $m\angle 3$, $m\angle 4$, and $m\angle 5$.

Parallel Lines Practice : In-Class Work

1
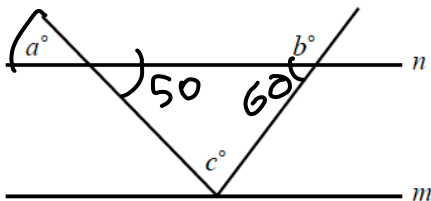

Note: Figure not drawn to scale

In the figure above, $r \parallel t$. What is the value of $x + y$?

A) 37

B) 40

C) 43

D) 46

(handwritten) $5x + 4 + 3x = 180$
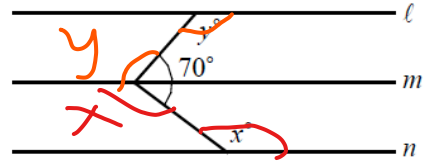
$8x = 176$

$x = 22$

$y = 15$

2

In the figure above, $m \parallel n$. If $a = 50$ and $b = 120$, what is the value of c?

A) 50

B) 60

C) 70

D) 80

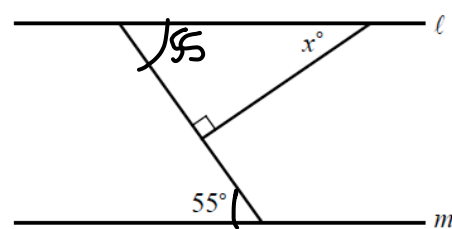*(handwritten)* $180 - 110$

3

Note: Figure not drawn to scale.

In the figure above, lines ℓ, m, and n are parallel. What is the value of $x + y$?
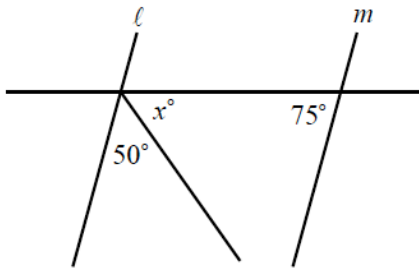
A) 160

B) 200

C) 230

D) 290

(handwritten) $x + y + 70 = 360$

4

In the figure above, $\ell \parallel m$. What is the value of x?

A) 30

B) 35

C) 40

D) 45

(handwritten) $55 + 90 + x = 180$
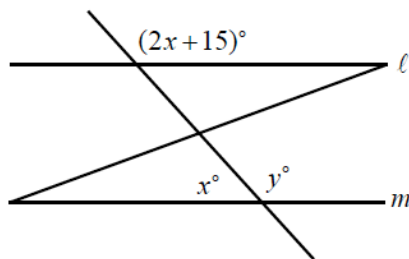
Quiz – Lines & Angles Homework

1

Note: Figure not drawn to scale.

In the figure above, $\ell \parallel m$. What is the value of x?
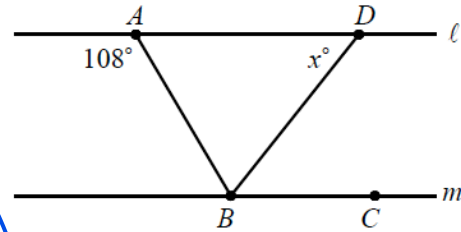
A) 45

B) 50

C) 55

D) 60

2

Note: Figure not drawn to scale.

In the figure above, $\ell \parallel m$. What is the value of y?

A) 120

B) 125

C) 130

D) 135

3

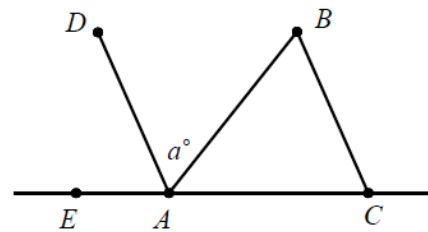Note: Figure not drawn to scale.

In the figure above, lines ℓ and m are parallel and $\overline{BD}$ bisects $\angle ABC$. What is the value of x?

A) 54
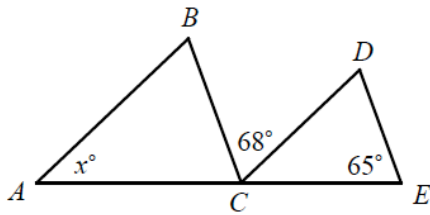
B) 60

C) 68

D) 72

4

In the figure above, $\overline{DA} \parallel \overline{BC}$ and $\overline{AB}$ bisects $\angle DAC$. What is the measure of $\angle BCA$ in terms of a?

A) $180 - a$

B) $2a - 180$

C) $180 - 2a$

D) $2a - 90$

5

Note: Figure not drawn to scale.

In the figure above, $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{DE}$. What is the value of x ?

A) 47

B) 51

C) 55

D) 57

6
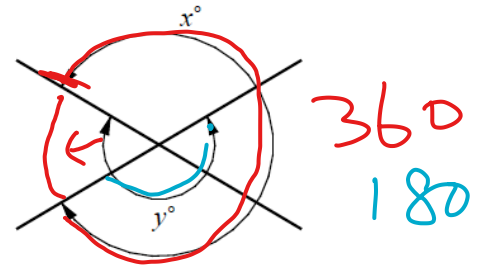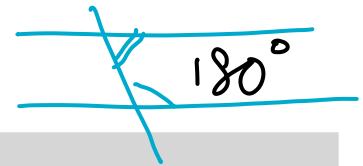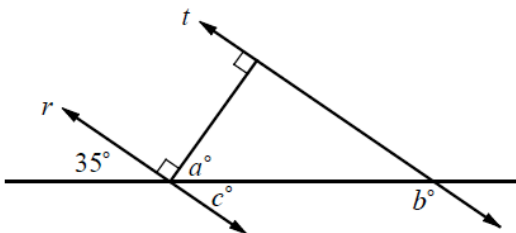
In the figure above, $r \parallel t$. What is the value of $a+b$?

A) 160

B) 175

C) 185

D) 200

7

In the figure above, what is the value of $x+y$?

8
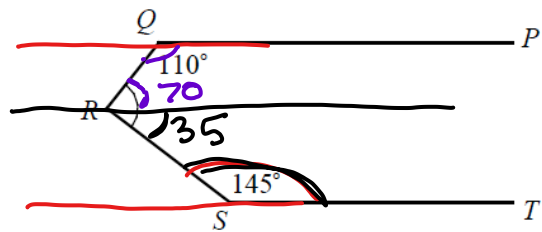
Note: Figure not drawn to scale.

In the figure above, $\overline{PQ}$ is parallel to $\overline{ST}$. What is the measure of $\angle QRS$?

Triangles

- Classifying triangles by angles: acute, right, obtuse

- Classifying triangles by sides: scalene, isoseles, equilateral

- Angle Sum Theorem:

- Exterior angle theorem: $a+b$

- Isosceles triangle theorem:

- Isosceles triangles inverse theorem:

- Equilateral triangle theorem:

$50°$

EXAMPLE 1 ✏

a. In $\triangle ABC$ shown below, $AB = BC$, $m\angle BCD = 110$ and $m\angle BDE = 140$.
   Find $m\angle 1$, $m\angle 2$, $m\angle 3$, and $m\angle 4$.
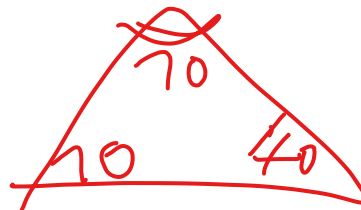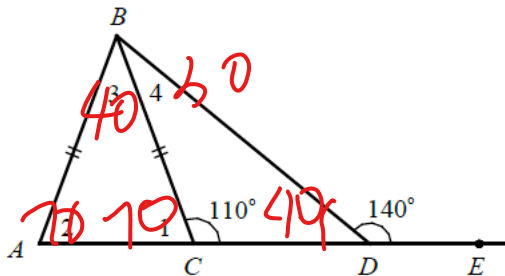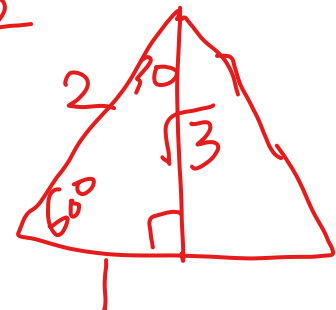
- Pythagorean Theorem: $a^2 + b^2 = c^2$

- 30-60-90 special right triangle:

- 45-45-90 special right triangle:

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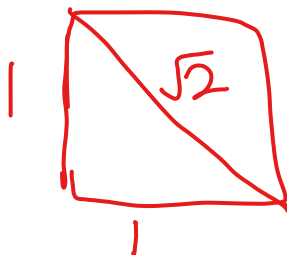
EXAMPLE 2 ✏

□ In the figure below, find the value of x.
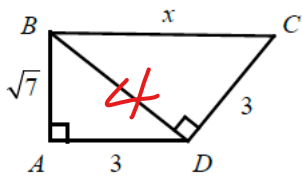
$\overline{BD} = \sqrt{7 + 3^2} = \sqrt{16} = 4$

$x = 5$

EXAMPLE 3 ✏

□ In the figures below, find the values of x and y.

a.

$2\sqrt{2}$

b.

$y = \dfrac{6}{\sqrt{3}}$

$x = \dfrac{3}{\sqrt{3}}$
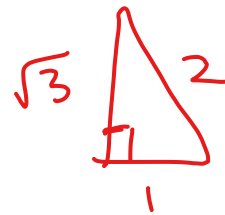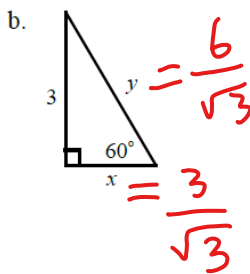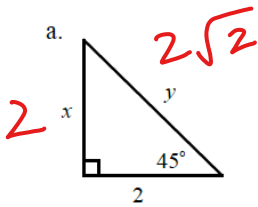
- AA similarity postulate:
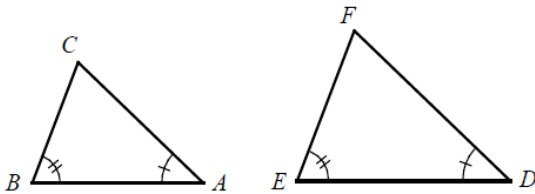
If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\angle C \cong \angle F$.
Therefore $\triangle ABC \sim \triangle DEF$, and
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF}.$$

- Triangle proportionality theorem:
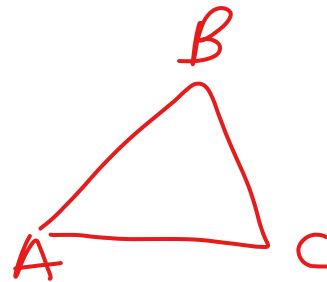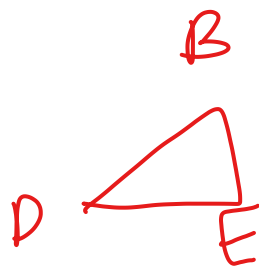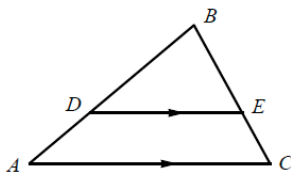
$\dfrac{\overline{BD}}{\overline{BE}} = \dfrac{\overline{BA}}{\overline{BC}}$

$\dfrac{\overline{BC}}{\overline{AC}} = \dfrac{\overline{BE}}{\overline{DE}}$

EXAMPLE 4 ✏️

□ In the figure below, $\overline{AB} \parallel \overline{DE}$. Find CD and CE.

$$\frac{9}{6} = \frac{6.75}{x}$$

$$\frac{9}{13} = \frac{6.75}{y}$$

$$x = 4.5$$
$$y = 9.75$$
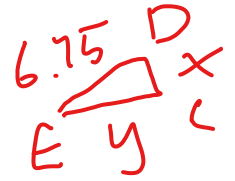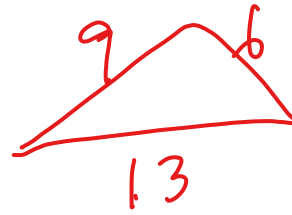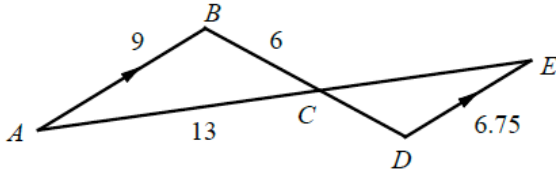
EXAMPLE 5 ✏️

In the figure at right, $\overline{AC} \parallel \overline{DE}$.
Find the length of $\overline{DA}$ and $\overline{AC}$.

$$\frac{6}{8} = \frac{x}{20}$$
$$x = 15$$
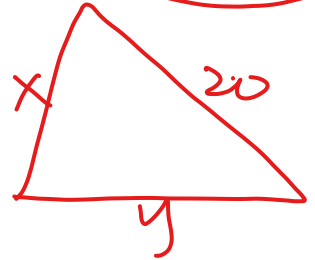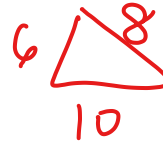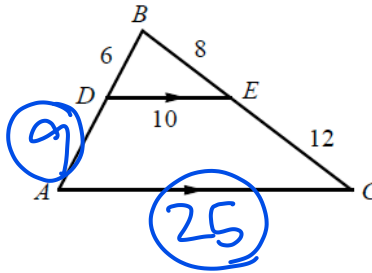
$$\frac{8}{10} = \frac{20}{y}$$
$$y = 25$$

Area of a triangle:

$$A = \frac{1}{2} b \cdot h$$

Area of a equilateral triangle:

$$A = \frac{s^2 \cdot \sqrt{3}}{4}$$

EXAMPLE 6 ✏️

□ In the figure below, if $AC = 6$, $BD = 4$, and $AB = 8$, what is the length of CE?
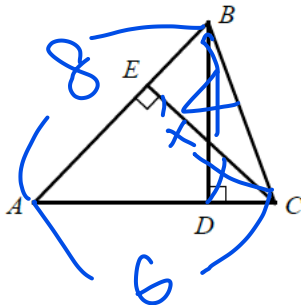
$$A = \frac{1}{2} \cdot 6 \cdot 4$$

$$A = \frac{1}{2} \cdot 8 \cdot x$$

$$x = 3$$

Triangles Practice: In-Class Work

1
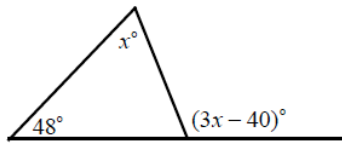
In the triangle above, what is the value of x?

A) 44

B) 48

C) 52

D) 56

2
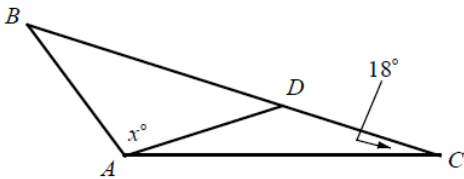
In $\triangle ABC$ above, if $AB = AD = DC$, what is the value of x?

A) 92

B) 96

C) 102

D) 108

3
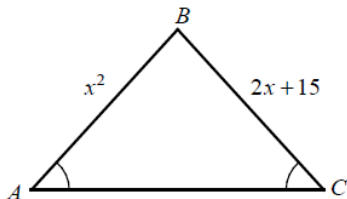
In $\triangle ABC$ above, $m\angle A = m\angle C$. If $x > 0$, what is the value of x?

4
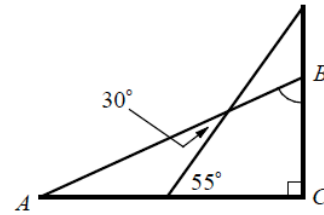
Note: Figure not drawn to scale.

In the figure above, $AC \perp BC$. What is the measure of $\angle ABC$?

A) 50

B) 55

C) 60

D) 65

5
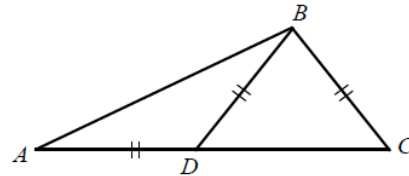
In the figure above, $AD = BD = BC$. If $m\angle A = 26$, what is the measure of $m\angle DBC$?

A) 68

B) 72

C) 76

D) 82

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1
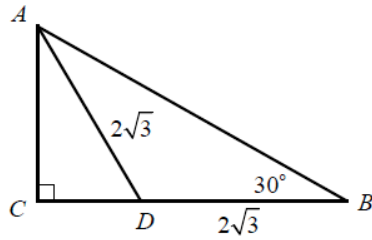
In the figure above, if $AD = BD = 2\sqrt{3}$, what is the length of AB?

A) $4\sqrt{3}$

B) $3\sqrt{6}$

C) 6

D) $6\sqrt{2}$

2
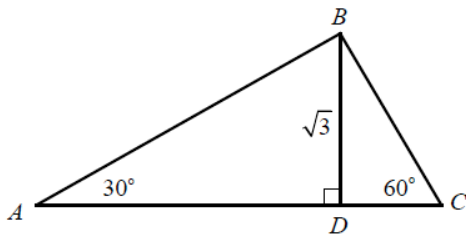
In $\triangle ABC$ above, $BD = \sqrt{3}$. What is the perimeter of $\triangle ABC$?

A) $2\sqrt{2} + 6$

B) $2\sqrt{3} + 6$

C) $2\sqrt{6} + 6$

D) $3\sqrt{2} + 6$

3
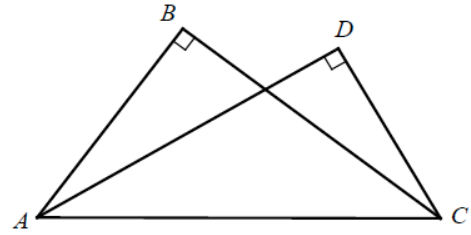
Note: Figure not drawn to scale.

In the figure above, $AB = 6$, $BC = 8$, and $CD = 5$. What is the length of AD?

A) $4\sqrt{3}$

B) $5\sqrt{2}$

C) $5\sqrt{3}$

D) $6\sqrt{2}$

4
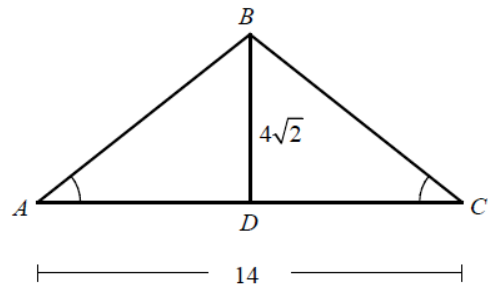
Note: Figure not drawn to scale.

In the figure above, $\angle A \cong \angle C$ and $\overline{BD}$ bisects $\overline{AC}$. What is the perimeter of $\triangle ABC$?

A) 32

B) 36

C) $14 + 10\sqrt{2}$

D) $14 + 12\sqrt{2}$

1
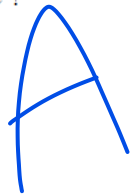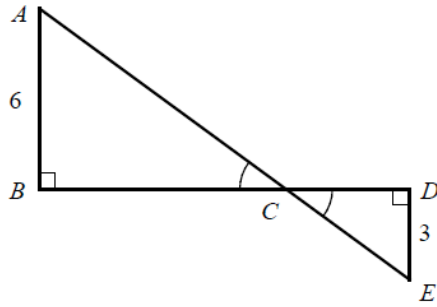
In the figure above, if $AB = 6$, $DE = 3$, and $BD = 12$, what is the length of AE?

A) 12

B) $9\sqrt{2}$

C) $8\sqrt{3}$

D) 15

2
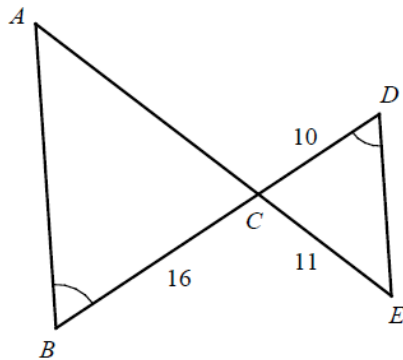
Note: Figure not drawn to scale.

In the figure above, $\angle B \cong \angle D$. If $BC = 16$, $CD = 10$, and $CE = 11$, what is the length of AE?

A) 16.8

B) 17.2

C) 17.6

D) 18.4

Questions 3 and 4 refer to the following information.
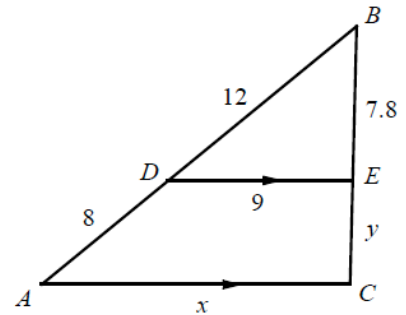
In the figure above, $\overline{DE} \parallel \overline{AC}$.

3

What is the value of x?

A) 12.5

B) 15

C) 16.5

D) 18

4

What is the value of y?

A) 5.2

B) 5.6

C) 6.0

D) 6.4

Triangles Quiz – Homework

1

In the figure above, the area of right triangle ABC is 60. What is the perimeter of $\triangle ABC$?

A) 34

B) 36

C) 38

D) 40

2

In triangle ABC above, if BD was increased by 50 percent and AC was reduced by 50 percent, how would the area of $\triangle ABC$ change?

A) The area of $\triangle ABC$ would be decreased by 25 percent.

B) The area of $\triangle ABC$ would be increased by 25 percent.

C) The area of $\triangle ABC$ would not change.

D) The area of $\triangle ABC$ would be decreased by 50 percent.

3
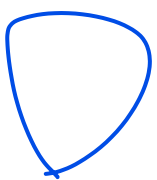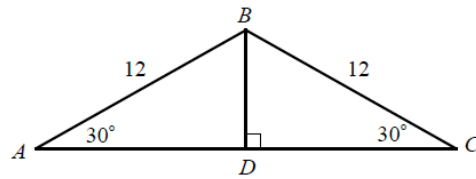
In the figure above, what is the area of $\triangle ABC$?

A) $24\sqrt{3}$

B) $30\sqrt{3}$

C) $36\sqrt{3}$

D) $48\sqrt{3}$

4
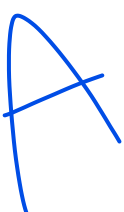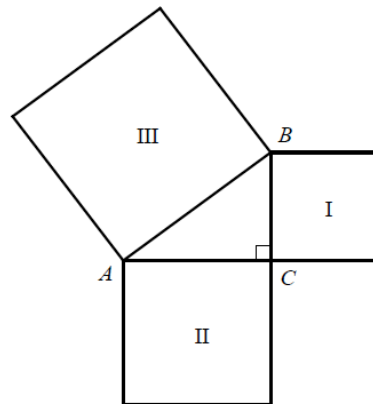
The figure above shows right triangle $\triangle ABC$ and three squares. If the area of square region I is 80 square inches and the area of square region II is 150 square inches, which of the following is true about the area of square region III?

A) Less than 230 square inches.

B) More than 230 square inches.

C) Equal to 230 square inches.

D) It cannot be determined from the information given.

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1
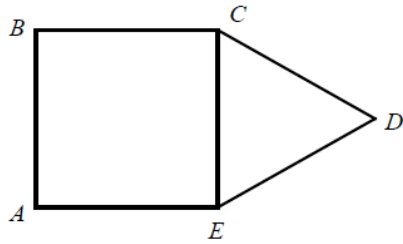
In the figure above, CDE is an equilateral triangle and $ABCD$ is a square with an area of $4x^2$. What is the area of triangle CDE in terms of x?

A) $\dfrac{\sqrt{3}}{2}x^2$

B) $\sqrt{3}x^2$

C) $\dfrac{3\sqrt{3}}{2}x^2$

D) $2\sqrt{3}x^2$

2
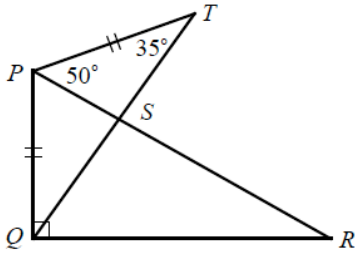
In the figure above, $\overline{PQ} \perp \overline{QR}$ and $\overline{PQ} \cong \overline{PT}$. What is the measure of $\angle R$?

A) 30

B) 35

C) 40

D) 45

3
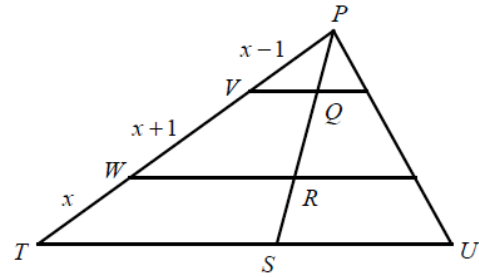
Note: Figure not drawn to scale.

In the figure above, $\overline{VQ} \parallel \overline{WR} \parallel \overline{TS}$.
If $PS = 15$, what is the length of $\overline{RS}$?

A) 4.5

B) 5

C) 6

D) 6.5

4
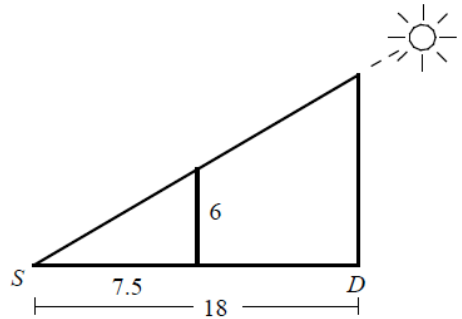
Note: Figure not drawn to scale.

A person 6 feet tall stands so that the ends of his shadow and the shadow of the pole coincide. The length of the person's shadow was measured 7.5 feet and the length of the pole's shadow, SD, was measured 18 feet. How tall is the pole?

A) 12.8

B) 13.6

C) 14.4

D) 15.2

5
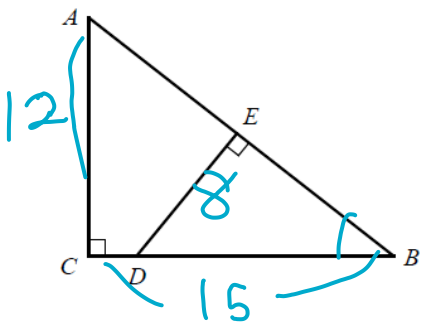
In the figure above, $\triangle ABC$ and $\triangle DBE$ are right triangles. If $AC = 12$, $BC = 15$, and $DE = 8$, what is the length of BE ?

A) 8.5
B) 9
C) 9.5
D) 10

6
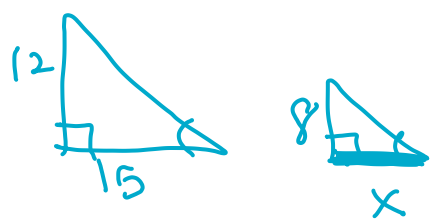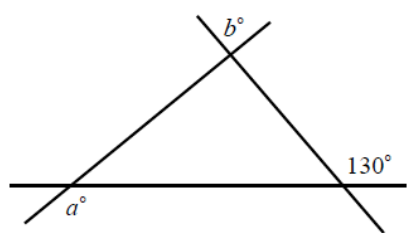
In the figure above, what is the value of $a - b$?

A) 50
B) 55
C) 60
D) 65

7
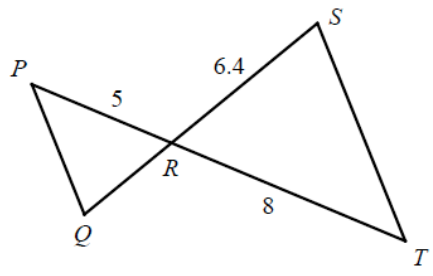
In the figure above, $\overline{PQ} \parallel \overline{ST}$ and segment PT intersects segment QS at R. What is the length of segment QS ?

8
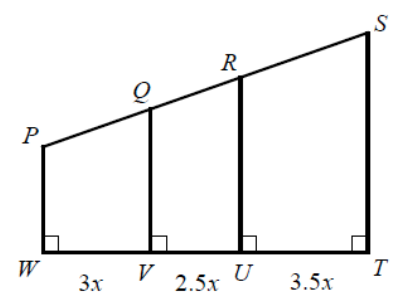
In the figure above, if $PS = 162$, what is the length of segment QR ?

9
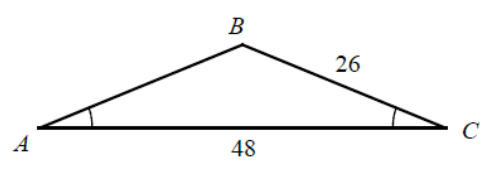
In the figure above, what is the area of the isosceles triangle ABC ?

Angles/Triangles - Homework
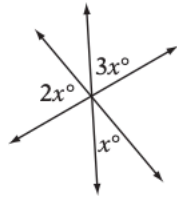
Note: Figure not drawn to scale.

1. The figure above shows the intersection of three lines. $x =$

   (A) 16
   (B) 20
   (C) 30
   (D) 60
   (E) 90
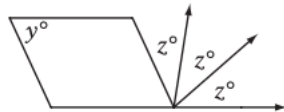
2. The figure above shows a parallelogram with one side extended. If $z = 40$, then $y =$

   (A) 40
   (B) 60
   (C) 80
   (D) 110
   (E) 120
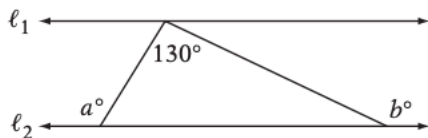
Note: Figure not drawn to scale.

3. In the figure above, if $\ell_1 \parallel \ell_2$, then $a + b =$

   (A) 130
   (B) 270
   (C) 280
   (D) 290
   (E) 310
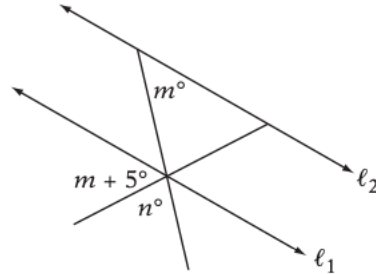
4. In the figure above, if $l_1 \parallel l_2$, then what is the value of n in terms of m?

   (A) $355 - 2m$
   (B) $185 - 2m$
   (C) $175 - 2m$
   (D) $95 - 2m$
   (E) $85 - 2m$
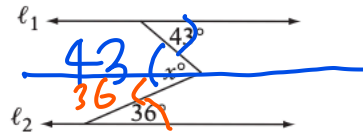
5. In the figure above, if $l_1 \parallel l_2$, then $x =$

   (A) 43
   (B) 69
   (C) 79
   (D) 101
   (E) 111
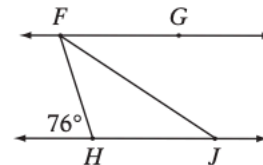
6. In the figure above, if $\overleftrightarrow{FG} \parallel \overleftrightarrow{HJ}$ and $\overline{FJ}$ bisects $\angle HFG$, what is the measure of $\angle FJH$?

   (A) 14
   (B) 38
   (C) 40
   (D) 56
   (E) 76
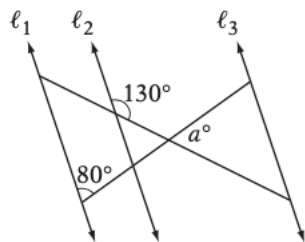
Note: Figure not drawn to scale.

7. In the figure above, if $\ell_1 \parallel \ell_2$ and $\ell_2 \parallel \ell_3$, then $a =$

(A) 50
(B) 55
(C) 60
(D) 65
(E) 70
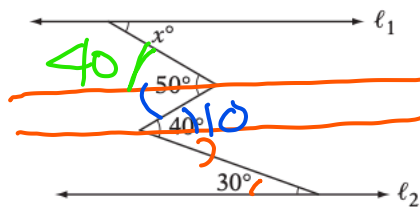
Note: Figure not drawn to scale.

8. In the diagram above, if $\ell_1 \parallel \ell_2$, then $x =$

(A) 65
(B) 60
(C) 50
(D) 45
(E) 40

C

x°

B

50°

A   *D*

A
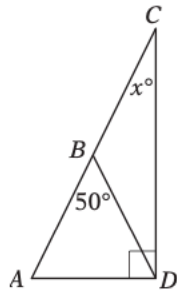
1. In the figure above, if $AB = BD$, then $x =$

(A) 25
(B) 30
(C) 35
(D) 50
(E) 65
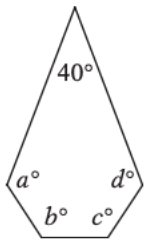
40°

$(n-2) \cdot 180$

a°   *d*°

b°   *c*°

540

50 D

2. In the figure above, $a + b + c + d =$

3. The three sides of a triangle have lengths of 9, 16, and k. Which of the following could equal k?

    I.   6
   II.   16
  III.   25

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, and III

9   16

$7 < k < 25$

$\frac{9 \quad 16}{25}$

a°

b°
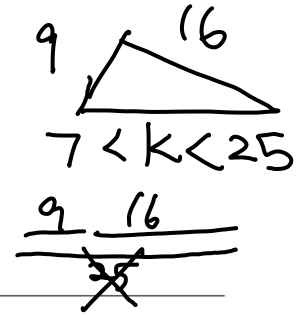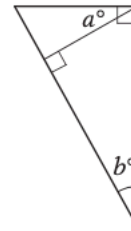
Note: Figure not drawn to scale.

4. Which of the following statements about a and b in the figure above must be true?

    I. $a = b$
   II. $a + b = 90$
  III. $a < 60$

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, and III

A

B

x°   100°   *y*°
A   *D*   *C*
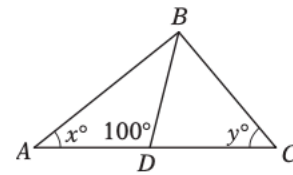
5. In the figure above, if $AD = DB = DC$, then $x + y =$

(A) 70
(B) 80
(C) 90
(D) 100
(E) 120
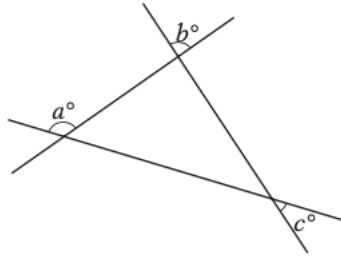
6. In the figure above, which of the following expresses a in terms of b and c?

(A) $180 - (b + c)$
(B) $180 - (b - c)$
(C) $90 - (b + c)$
(D) $90 - (b - c)$
(E) $b + c$
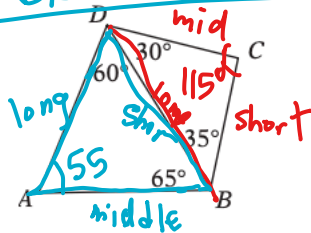
Note: Figure not drawn to scale.

7. Which of the following represents the correct ordering of the lengths of the five segments in the figure above?

(A) $AD > AB > DB > BC > DC$
(B) $AD > DB > AB > DC > BC$
(C) $AD > DB > AB > BC > DC$
(D) $AD > AB > DB > DC > BC$
(E) $AD > DB > DC > AB > BC$
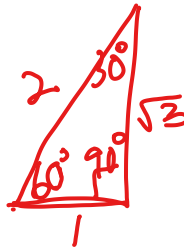
8. A triangle has two sides of lengths 4 centimeters and 6 centimeters. Its area is n square centimeters, where n is a prime number. What is the greatest possible value of n?

(A) 11
(B) 12
(C) 19
(D) 23
(E) 24

Day 17. 7/28 Wednesday

Circles
Finish pages 23-43 by 7/29

Circles

- Cord:

- Major arc / minor arc:

- Circumference= $2\pi r = d\pi$

- Arc length= $2\pi r \cdot \dfrac{\theta}{360°}$ $\qquad \dfrac{arc}{2\pi r} = \dfrac{\theta}{360}$

- Area of a circle= πr^2

- Area of a sector= $\pi r^2 \cdot \dfrac{\theta}{360°}$

- Secant line:

- Tangent line:

$\overline{AP} \perp r$
$\overline{BP} \perp r$
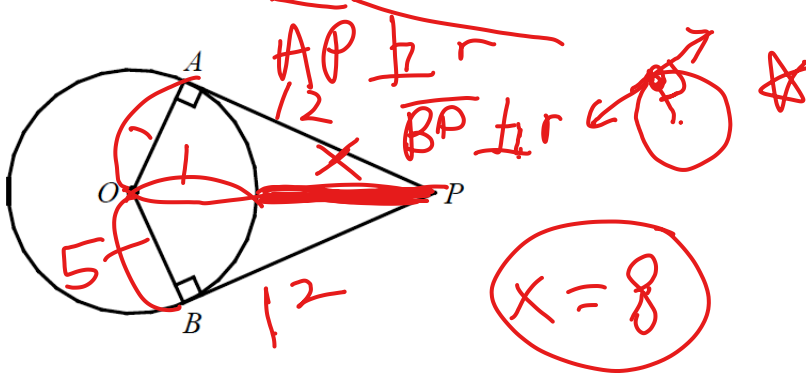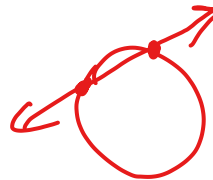
12 x 5 12 $\boxed{x = 8}$

EXAMPLE 1 ✏️

In $\odot O$ shown at the right, $\overline{PA}$ and $\overline{PB}$ are tangents, $PB = 12$, and $m\angle APO = 25$.

a. Find the measure of $\angle POA$. $180 - 90 - 25 = 65°$
b. Find the length of PA. 12
c. Find the radius of $\odot O$.

A 12 r $25°$ O P r $25°$ 12 B
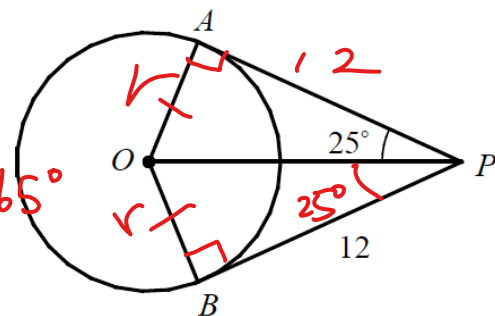
$$\tan 25° = \frac{r}{12} \qquad r = 5.6$$

EXAMPLE 2 ✏

□ In circle O shown at the right, $\overline{AB}$ is tangent to the circle.

a. Find the area of the shaded region.

b. Find the perimeter of the shaded region.
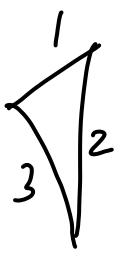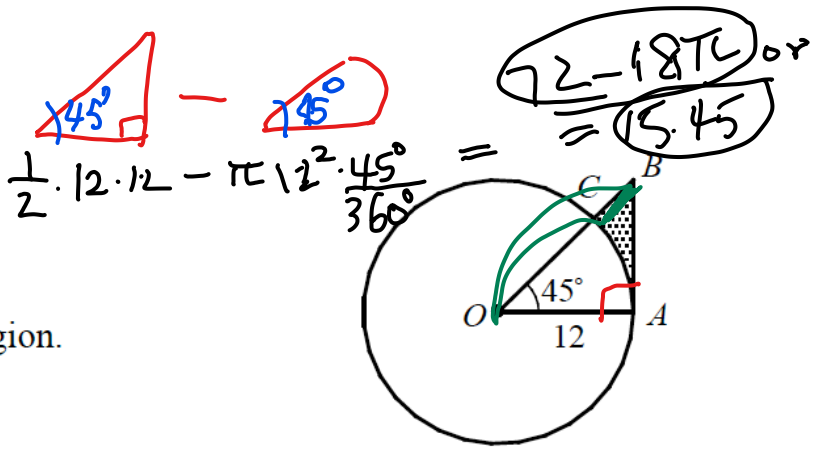
$$\frac{1}{2} \cdot 12 \cdot 12 - \pi \cdot 12^2 \cdot \frac{45°}{360°} = $$

$$72 - 18\pi \text{ or } = 15.45$$

$$/ = 12\sqrt{2} - 12$$

$$/ = 12$$
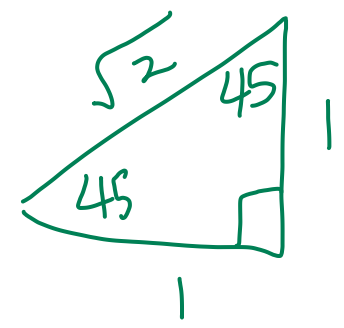
$$) = 2\pi \cdot 12 \cdot \frac{45}{360} = 3\pi$$

$$\boxed{12\sqrt{2} + 3\pi}$$

EXAMPLE 3 ✏

□ The radius of a bicycle wheel is 12 inches. What is the number of revolutions the wheel makes to travel 1 mile? (1 mile $= 5,280$ ft)

$$\text{distance} = 2\pi r \cdot \#\text{rev}$$

$$5280 \cdot 12 = 2\pi \cdot 12 \cdot n$$

$$n = 840$$

Circles – Homework

In the figure above, $\overline{BD}$ is a diameter, and $\overline{PA}$ and $\overline{PD}$ are tangents to circle O. $m\angle CDE = 52$, $m\angle APD = 45$, and $AP = 9$.

1

What is the measure of $\angle ODC$?

38

2

What is the measure of $\angle OCD$?

38

3

What is the measure of $\angle AOD$?

135

4

What is the length of PD?

9

5

In the figure above, $\odot O$ is inscribed in ΔPQR. If $PA = 12$, $QA = 6$, and $RB = 9.5$, what is the perimeter of ΔPQR?

A) 46
B) 49
C) 52
D) 55

D

6

In the figure above, $\overline{OP}$ is a radius and $\overline{PQ}$ is tangent to circle O. If the radius of circle O is 10 and $QR = 16$, what is the length of $\overline{PQ}$?

C

A) 16
B) 20
C) 24
D) 28

Questions 1 - 4 refer to the following information.
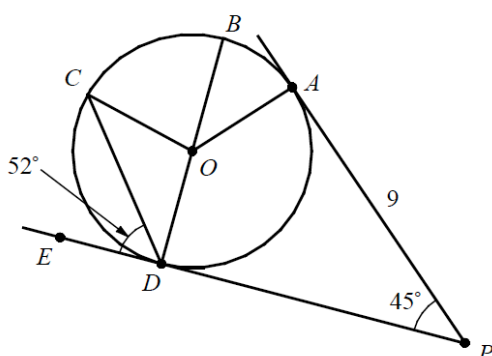
Same page (handwritten)

In the figure above, $\overline{BD}$ is a diameter, and $\overline{PA}$ and $\overline{PD}$ are tangents to circle O. $m\angle CDE = 52$, $m\angle APD = 45$, and $AP = 9$.

1

What is the measure of $\angle ODC$?

2

What is the measure of $\angle OCD$?

3

What is the measure of $\angle AOD$?

4

What is the length of PD ?

5
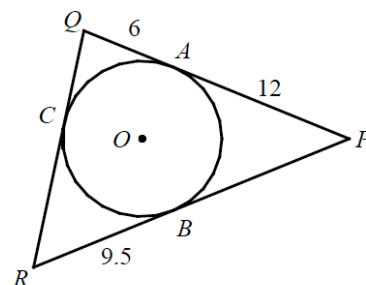
In the figure above, $\odot O$ is inscribed in $\triangle PQR$. If $PA = 12$, $QA = 6$, and $RB = 9.5$, what is the perimeter of $\triangle PQR$?

A) 46

B) 49

C) 52

D) 55

6
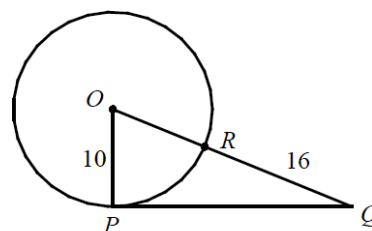
In the figure above, $\overline{OP}$ is a radius and $\overline{PQ}$ is tangent to circle O. If the radius of circle O is 10 and $QR = 16$, what is the length of $\overline{PQ}$?

A) 16

B) 20

C) 24

D) 28

Circles in the Coordinate Plane

- Standard equation of a circle with center (h,k) and radius r is : $(x-h)^2 + (y-k)^2 = r^2$

Teacher Modeled	Student Try it!
What is an equation of circle *O* shown in the graph below?	What is an equation of the circle shown in the graph below?

Teacher Modeled:
1) $(x+1)^2 + (y-3)^2 = 25$
2) $(x-1)^2 + (y+3)^2 = 25$
3) $(x-5)^2 + (y+6)^2 = 25$
4) $(x+5)^2 + (y-6)^2 = 25$
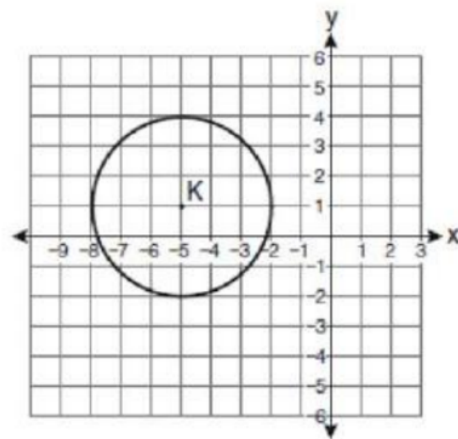
Student Try it!:
1) $(x+5)^2 + (y-1)^2 = 3$
2) $(x+5)^2 + (y-1)^2 = 9$
3) $(x-5)^2 + (y+1)^2 = 3$
4) $(x-5)^2 + (y+1)^2 = 9$

EXAMPLE 1 ✏️

Write an equation of a circle with center (–3, 2) and r = 2 .

$$(x+3)^2 + (y-2)^2 = 4$$

EXAMPLE 2 ✏️

Write an equation of a circle that is tangent to the y- axis and has center (4,3) .

$$(x-4)^2 + (y-3)^2 = 16$$
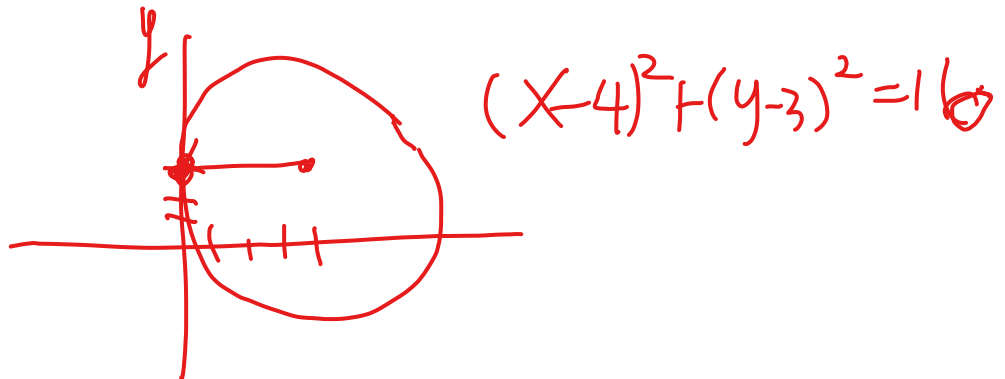
EXAMPLE 3 ✏️

Write an equation of a circle whose endpoints of its diameter are at (−4,8) and (2,−4) .

$$c: (-1, 2)$$

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$r = \sqrt{9 + 36} = \sqrt{45}$$

$$(x+1)^2 + (y-2)^2 = 45$$

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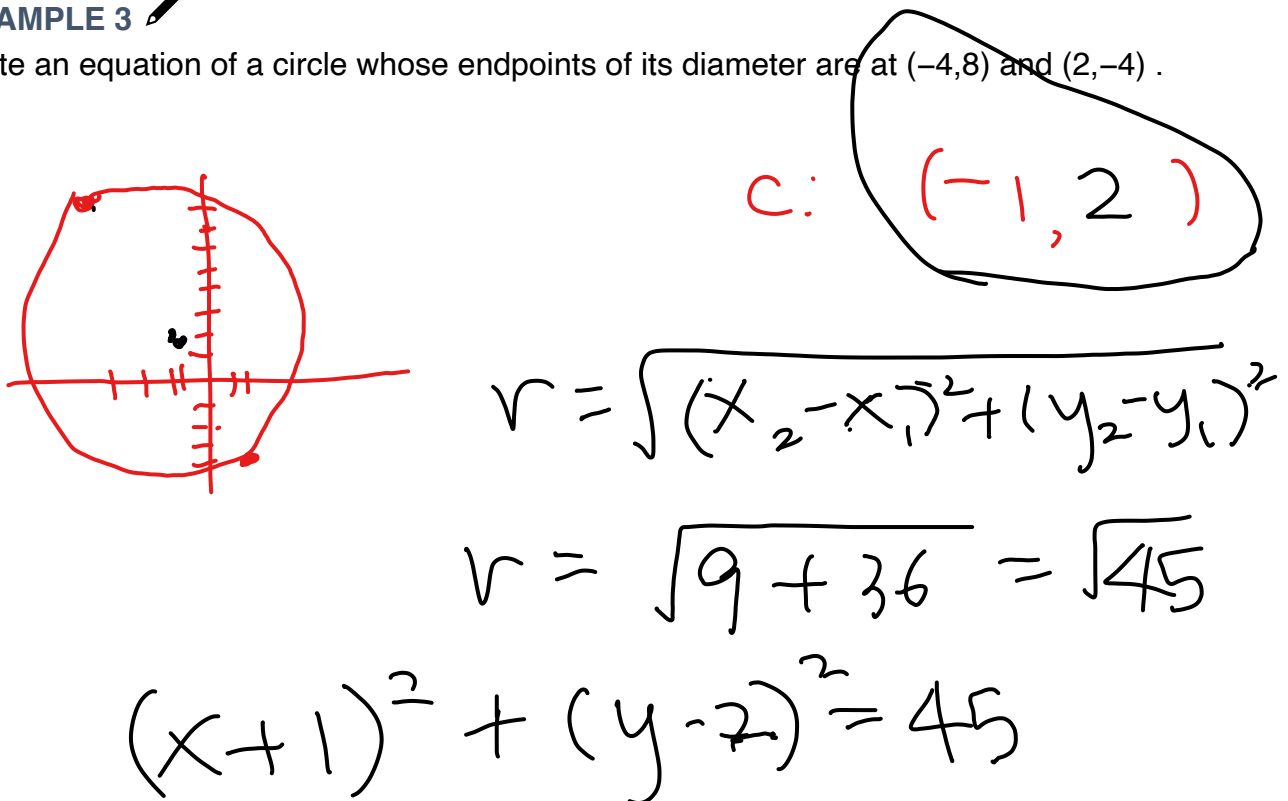
Circles - Completing the square

Consider, for example, the process of rewriting the expanded equation
$x^2 + y^2 + 18x + 14y + 105 = 0$ in standard form:

$$x^2 + y^2 + 18x + 14y + 105 = 0$$

$$x^2 + y^2 + 18x + 14y = -105$$

$$\underbrace{(x^2 + 18x)}_{} + \underbrace{(y^2 + 14y)}_{} = -105$$

$$(x^2 + 18x + 81) + (y^2 + 14y + 49) = -105 + 81 + 49$$

$$(x + 9)^2 + (y + 7)^2 = 25$$

$$(x - (-9))^2 + (y - (-7))^2 = 5^2$$

Now we can tell that the center of the circle is $(-9, -7)$ and the radius is 5.

Handwritten annotations:

$$\text{vertex} = \left(-\frac{b}{2a}, \right)$$

$$C = \left(\frac{b}{2}\right)^2$$

$+C$ (over $x^2 + 18x$), $+C$ (over $y^2 + 14y$)

EXAMPLE ✏️

Find the center and radius of a circle with the equation
$x^2 + y^2 - 10x - 16y + 53 = 0$

What is the center of this circle?

$(\boxed{5}, \boxed{8})$

What is the radius of this circle?

$\boxed{6}$ units

Handwritten work:

$$(x^2 - 10x + 25) + (y^2 - 16y + 64) = -53 + 25 + 64$$

$$(x - 5)^2 + (y - 8)^2 = 36$$

Completing the square – Practice Homework

: Graph the equation: $x^2 + y^2 - 2x + 4y - 4 = 0$

$$(x-1)^2 + (y+2)^2 = 3^2$$
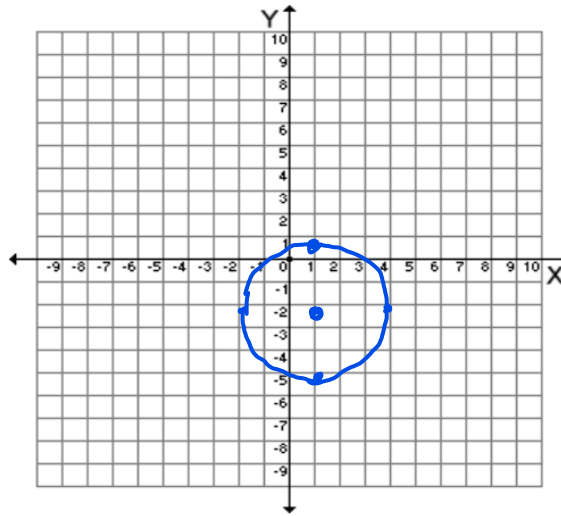
: Graph: $x^2 + y^2 + 6x - 2y + 1 = 0$

$$(x+3)^2 + (y-1)^2 = 3^2$$

: Graph: $x^2 + y^2 + x - y - ½ = 0$

$$\left(x+\tfrac{1}{2}\right)^2 + \left(y-\tfrac{1}{2}\right)^2 = 1$$

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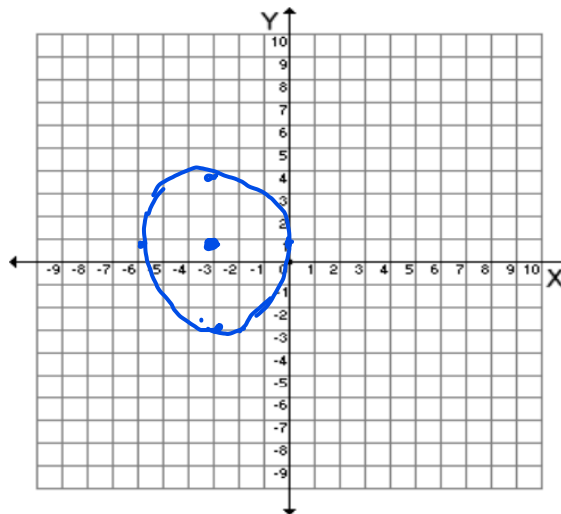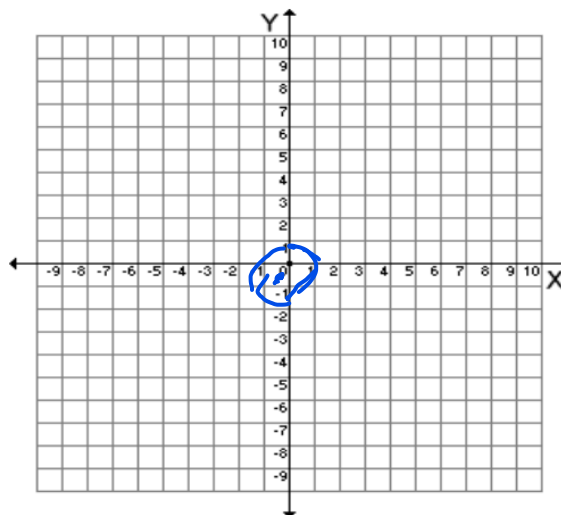
Circles in Coordinate - Homework

1
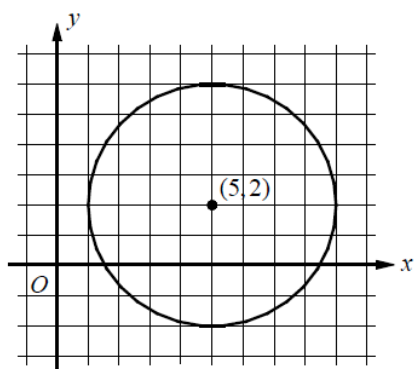
(5,2)

Which of the following equations represents the equation of the circle shown in the xy- plane above?

A) $(x+5)^2 +(y+2)^2 = 4$

B) $(x-5)^2 +(y-2)^2 = 4$

C) $(x+5)^2 +(y+2)^2 = 16$

D) $(x-5)^2 +(y-2)^2 = 16$

2

Which of the following is an equation of a circle in the xy- plane with center $(-2,0)$ and a radius with endpoint $(0,\frac{3}{2})$?

A) $x^2 +(y-\frac{3}{2})^2 = \frac{5}{2}$

B) $x^2 +(y-\frac{3}{2})^2 = \frac{25}{4}$

C) $(x+2)^2 + y^2 = \frac{25}{4}$

D) $(x-2)^2 + y^2 = \frac{25}{4}$

3

$$x^2 +12x+ y^2 -4y+15 = 0$$

The equation of a circle in the xy- plane is shown above. Which of the following is true about the circle?

A) center $(-6,2)$, radius $= 5$

B) center $(6,-2)$, radius $= 5$

C) center $(-6,2)$, radius $= \sqrt{15}$

D) center $(6,-2)$, radius $= \sqrt{15}$

4

Which of the following represents an equation of a circle whose diameter has endpoints $(-8,4)$ and $(2,-6)$?

A) $(x-3)^2 +(y-1)^2 = 50$

B) $(x+3)^2 +(y+1)^2 = 50$

C) $(x-3)^2 +(y-1)^2 = 25$

D) $(x+3)^2 +(y+1)^2 = 25$

5

$$x^2 +2x+ y^2 -4y-9 = 0$$

The equation of a circle in the xy- plane is shown above. If the area of the circle is $k\pi$, what is the value of k ?

Circles -Quiz Homework

1
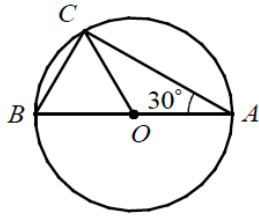
In the figure above, O is the center of the circle and $\overline{AB}$ is a diameter. If the length of $\overline{AC}$ is $4\sqrt{3}$ and $m\angle BAC = 30$, what is the area of circle O?

A) 12π

B) 16π

C) 18π

D) 24π

2
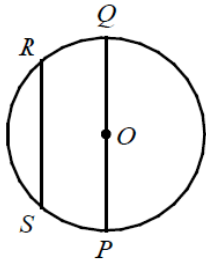
In the circle above, chord $\overline{RS}$ is parallel to diameter $\overline{PQ}$. If the length of $\overline{RS}$ is $\dfrac{3}{4}$ of the length of $\overline{PQ}$ and the distance between the chord and the diameter is $2\sqrt{7}$, what is the radius of the circle?

A) 6

B) $3\sqrt{7}$

C) 8

D) $4\sqrt{7}$

3
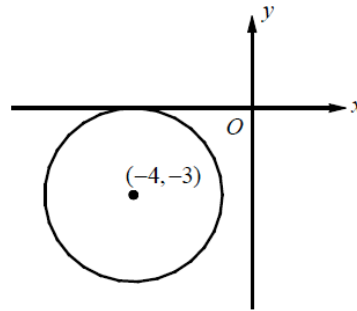
In the figure above, the circle is tangent to the x-axis and has center $(-4,-3)$. Which of the following equations represents the equation of the circle shown in the xy-plane above?

A) $(x+4)^2 + (y+3)^2 = 9$

B) $(x-4)^2 + (y-3)^2 = 9$

C) $(x+4)^2 + (y+3)^2 = 3$

D) $(x-4)^2 + (y-3)^2 = 3$

4
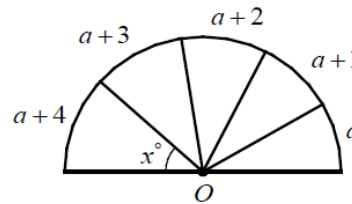
The figure above shows a semicircle with the lengths of the adjacent arcs a, $a+1$, $a+2$, $a+3$, and $a+4$. If the value of x is 42, what is the value of a?

A) 7

B) 8

C) 9

D) 10

5
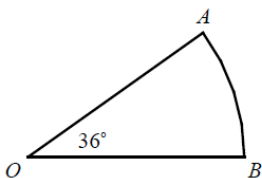
In the figure above, the length of arc $\overset{\frown}{AB}$ is π. What is the area of sector OAB?

A) 2π

B) $\dfrac{5}{2}\pi$

C) 3π

D) $\dfrac{7}{2}\pi$

6

$$x^2 - 4x + y^2 - 6x - 17 = 0$$

What is the area of the circle in the xy-plane above?

A) 20π

B) 24π

C) 26π

D) 30π

7

Which of the following is the equation of a circle that has a diameter of 8 units and is tangent to the graph of $y = 2$?

A) $(x+1)^2 + (y+2)^2 = 16$

B) $(x-1)^2 + (y-2)^2 = 16$

C) $(x+2)^2 + (y+1)^2 = 16$

D) $(x-2)^2 + (y-1)^2 = 16$

8
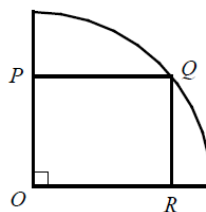
In the figure above, rectangle $OPQR$ is inscribed in a quarter circle that has a radius of 9. If $PQ = 7$, what is the area of rectangle $OPQR$?

A) $24\sqrt{2}$

B) $26\sqrt{2}$

C) $28\sqrt{2}$

D) $30\sqrt{2}$

9

In a circle with center O, the central angle has a measure of $\dfrac{2\pi}{3}$ radians. The area of the sector formed by central angle AOB is what fraction of the area of the circle?

10

A wheel with a radius of 2.2 feet is turning at a constant rate of 400 revolutions per minute on a road. If the wheel traveled $k\pi$ miles in one hour what is the value of k? (1 mile = 5,280 feet)

Radians

180 degrees = _____ π _____ radians

$90° = \dfrac{\pi}{2}$

$45° = \dfrac{\pi}{4}$

$540° = 3\pi$

In the figure below, the length of $\overline{PA}$ is 3 units.

306°

25

$\dfrac{3}{10}\pi$

54° =

$\dfrac{\pi}{3}$

$\dfrac{11\pi}{18}$

60°

3
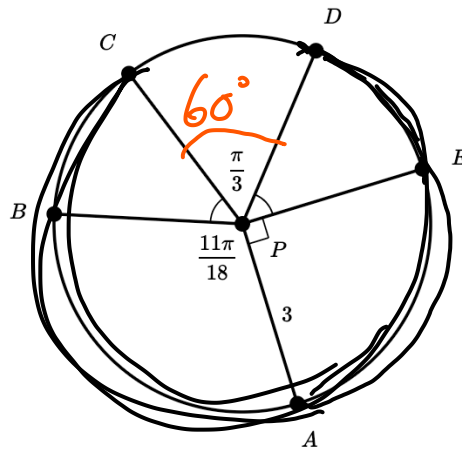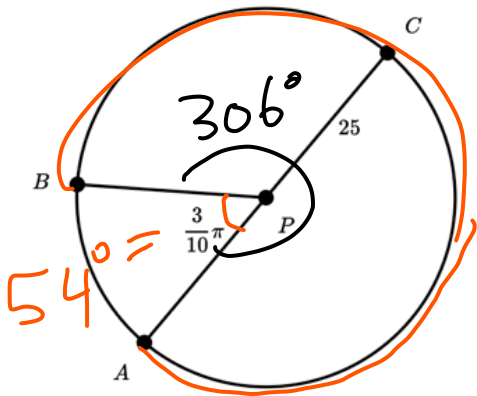
What is the exact length of $\overset{\frown}{BCA}$?

Choose 1 answer:

(A) $\dfrac{35}{2}\pi$

(B) $\dfrac{17}{10}\pi$

(C) $\dfrac{85}{2}\pi$

(D) $\dfrac{15}{2}\pi$

$2\pi r \cdot \dfrac{\theta}{360}$

$2\pi \cdot 25 \cdot \dfrac{306}{360}$

What is the exact length of $\overset{\frown}{DAC}$ on circle P?

Choose 1 answer:

(A) π

(B) 5π

(C) 6π

(D) $\dfrac{5}{3}\pi$

$2\pi r \cdot \dfrac{\theta}{360}$

$2\pi \cdot 3 \cdot \dfrac{300}{360}$

(Handwritten: 95° = and 19/36 π, π/3 = 60°)
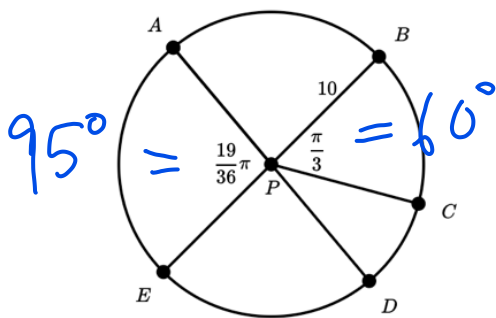
What is the exact length of $\overset{\frown}{CD}$?

Choose 1 answer:

(A) $\dfrac{95}{18}\pi$

(B) $\dfrac{35}{18}\pi$ *(circled)*

(C) $\dfrac{10}{3}\pi$

(D) $\dfrac{7}{36}\pi$

(Handwritten: $2\pi \cdot 10 \cdot \dfrac{35}{360}$)
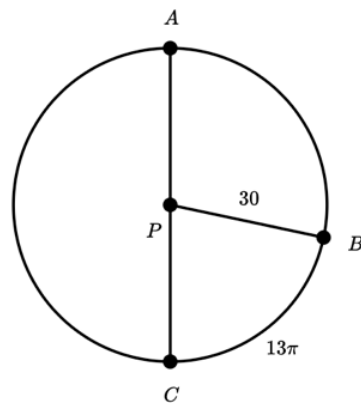
What is the arc measure of $\overset{\frown}{AB}$, in radians?

Choose 1 answer:

(A) $\dfrac{102}{30}\pi$

(B) $\dfrac{17}{30}\pi$

(C) 102π

(D) 17π *(circled)*

(Handwritten: $\left(2\pi \cdot 30 \cdot \dfrac{1}{2}\right) - 13\pi$)
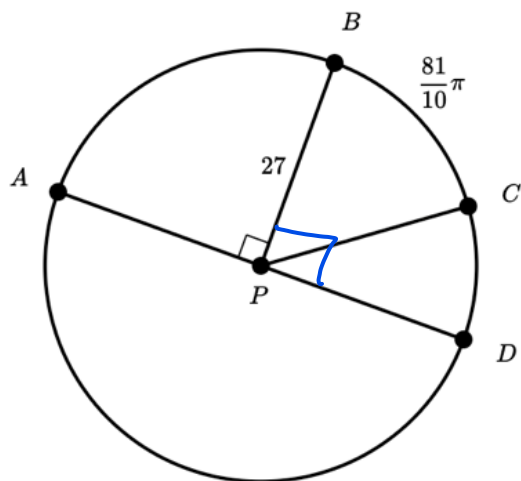
What is the arc measure of $\overset{\frown}{CD}$, in radians?

Choose 1 answer:

(A) $\dfrac{\pi}{5}$

(B) $\dfrac{3}{10}\pi$

(C) 36π

(D) $\dfrac{27}{5}\pi$ *(circled)*

(Handwritten: $2\pi \cdot 27 \cdot \dfrac{90}{360} - \dfrac{81}{10}\pi =$)

Right triangle Trigonometry
Soh Cah Toa

1. Find the missing side **'x'** to 1 decimal place.

[Triangle 1: right triangle with 60° angle, base 14, and side x]

[Triangle 2: right triangle with right angle at top, side 3.2, angle 25°, base x]
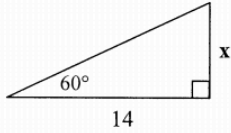
$$\tan 60° = \frac{x}{14}$$

$$x = 14 \cdot \tan 60°$$

$$x = 24.25$$
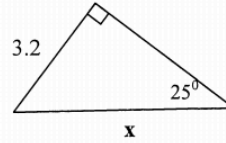
$$\sin 25° = \frac{3.2}{x}$$

$$x = \frac{3.2}{\sin 25°}$$

$$x = 7.57$$

2. Solve $\triangle XYZ$ where $<Z = 90°$, $z = 10$, & $y = 6$.   Answers to 1 decimal place.

[Triangle diagram labeled with 10, 6, X, Y, Z]
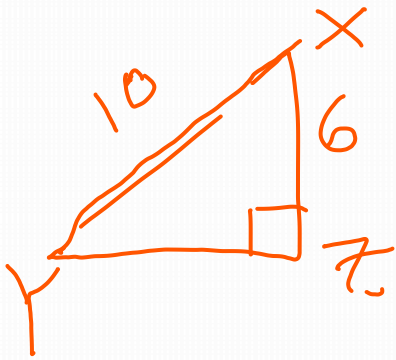
$$\overline{YZ} = 8$$

$$\sin \angle Y = \frac{6}{10}$$

$$\angle Y = \sin^{-1}\left(\frac{6}{10}\right) = 36.9°$$

$$\angle X = 90 - 36.9$$

$$= 53.1°$$

[Diagram with 8°, 108, x, and angle of depression]
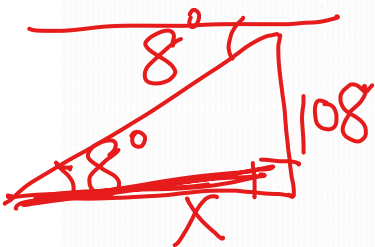
3. From the top of a cliff, 108m high, the angle of depression to a boat on the water is 8°. Determine the horizontal distance the boat is from the cliff. (answer to 1 decimal place) Clearly label the angle of depression.

$$\tan 8° = \frac{108}{x}, \quad x = \frac{108}{\tan 8} = 768.46$$

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Day 17 Homework

https://www.khanacademy.org/mission/sat/tasks/4757111756374016

https://www.khanacademy.org/mission/sat/tasks/5020489326346240

https://www.khanacademy.org/mission/sat/tasks/6341329309483008

https://www.khanacademy.org/math/trigonometry/trigonometry-right-triangles/test/trigonometry-right-triangles-unit-test?modal=1
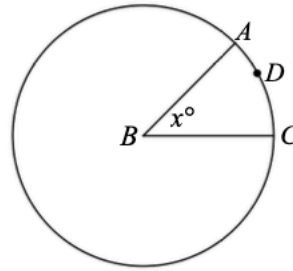
Note: Figure not drawn to scale.

4. In the figure above, the ratio of the circumference of circle B to the length of arc ADC is $8:1$. What is the value of x?

45

1. If the area of a circle is 64π, then the circumference of the circle is
   (A) 8π
   (B) 16π
   (C) 32π
   (D) 64π
   (E) 128π

B

5. In the figure above, if the area of the circle with center O is 100π and CA has a length of 6, what is the length of AB?
   (A) 2　　(B) 3　　(C) 4　　(D) 5　　(E) 6

A

2. If the minute hand of a clock moves 45 degrees, how many minutes of time have passed?
   (A) 6
   (B) 7.5
   (C) 15
   (D) 30
   (E) 36.5

B
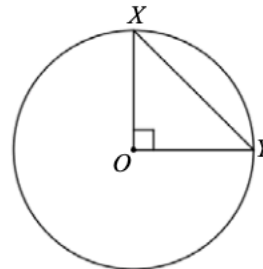
6. In the figure above, O is the center of the circle. If the area of triangle XOY is 25, what is the area of the circle?
   (A) 25π
   (B) $25\pi\sqrt{2}$
   (C) 50π
   (D) $50\pi\sqrt{3}$
   (E) 625π

C

3. If the circumference of a circle is 1, what is the radius of the circle?
   (A) $\dfrac{1}{2\pi}$
   (B) $\dfrac{1}{\pi}$
   (C) $\dfrac{1}{2}$
   (D) $\dfrac{\pi}{2}$
   (E) π

A
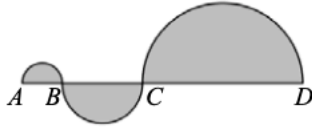
7. Each of the 3 shaded regions above is a semicircle. If $AB = 4$, $CD = 2BC$, and $BC = 2AB$, then the area of the entire shaded figure is
   (A) 28π
   (B) 42π
   (C) 84π
   (D) 96π
   (E) 168π
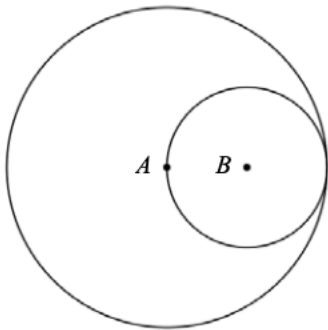
8. In the diagram above, if the circle with center A has an area of 72π, what is the area of the circle with center B?
   (A) 18π
   (B) 24π
   (C) 30π
   (D) 36π
   (E) 48π

9. If the diameter of a circle increases by 50 percent, by what percent will the area of the circle increase?
   (A) 25%
   (B) 50%
   (C) 100%
   (D) 125%
   (E) 225%

10. If an arc with a length of 12π is $\dfrac{3}{4}$ of the circumference of a circle, what is the shortest distance between the endpoints of the arc?
    (A) 4
    (B) $4\sqrt{2}$
    (C) 8
    (D) $8\sqrt{2}$
    (E) 16
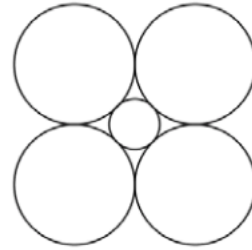
11. The total area of the four equal circles in the figure above is 36π, and the circles are all tangent to one another. What is the diameter of the small circle?
    (A) $6\sqrt{2}$
    (B) $6 + \sqrt{2}$
    (C) $3\sqrt{2} - 3$
    (D) $6\sqrt{2} - 6$
    (E) $6\sqrt{2} + 6$

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1. The length and width of a rectangle are in the ratio of 5:12. If the rectangle has an area of 240 square centimeters, what is the length, in centimeters, of its diagonal?

(A) 26
(B) 28
(C) 30
(D) 32
(E) 34

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2. A spider on a flat horizontal surface walks 10 inches east, then 6 inches south, then 4 inches west, then 2 inches south. At this point, how many inches is the spider from its starting point?

(A) 8
(B) 10
(C) 12
(D) 16
(E) 18

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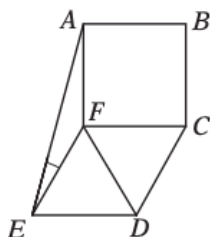
3. In the figure above, *ABCF* is a square and △*EFD* and △*FCD* are equilateral. What is the measure of ∠*AEF*?

(A) 15°
(B) 20°
(C) 25°
(D) 30°
(E) 35°

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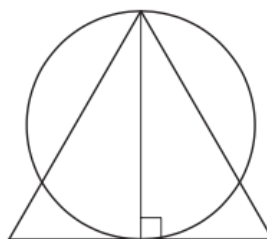
4. In the figure above, an equilateral triangle is drawn with an altitude that is also the diameter of the circle. If the perimeter of the triangle is 36, what is the circumference of the circle?

(A) $6\sqrt{2}\pi$

(B) $6\sqrt{3}\pi$

(C) $12\sqrt{2}\pi$

(D) $12\sqrt{3}\pi$

(E) 36π

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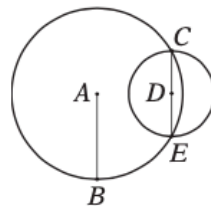
5. In the figure above, *A* and *D* are the centers of the two circles, which intersect at points *C* and *E*. $\overline{CE}$ is a diameter of circle *D*. If *AB* = *CE* = 10, what is *AD*?

(A) 5

(B) $5\sqrt{2}$

(C) $5\sqrt{3}$

(D) $10\sqrt{2}$

(E) $10\sqrt{3}$

6. Point H has coordinates $(2, 1)$, and point J has coordinates $(11, 13)$. If $\overline{HK}$ is parallel to the x-axis and $\overline{JK}$ is parallel to the y-axis, what is the perimeter of $\triangle HJK$?

36
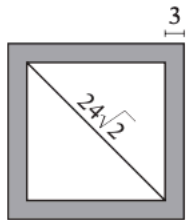
7. A square garden with a diagonal of length $24\sqrt{2}$ meters is surrounded by a walkway 3 meters wide. What is the area, in square meters, of the walkway?

324
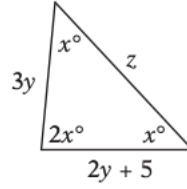
Note: Figure not drawn to scale.

8. In the figure above, what is the value of z?

(A) 15

(B) $15\sqrt{2}$

(C) $15\sqrt{3}$

B

(D) $30\sqrt{2}$

(E) $30\sqrt{3}$

1. If point A has coordinates $(3, 5)$, point B has coordinates $(3, 2)$, and $ABCD$ is a square, which of the following could be the coordinates of point C?

(A) $(4, 2)$ (B) $(6, 2)$ (C) $(6, 6)$
(D) $(4, 6)$ (E) $(8, 2)$

2. If ℓ_1 is a horizontal line passing through $(1, 8)$ and ℓ_2 is a vertical line passing through $(-3, 4)$, then at what point do ℓ_1 and ℓ_2 intersect?

(A) $(-3, 8)$ (B) $(1, 4)$ (C) $(-1, 6)$
(D) $(-2, 12)$ (E) $(0, 0)$

3. The point $(-3, 4)$ is on a circle with its center at the origin. Which of the following points must also be on the circle?
   I. $(0, -5)$
   II. $(-4, -3)$
   III. $(3, 4)$

(A) I only   (B) II only
(C) I and II only   (D) II and III only
(E) I, II, and III

4. If the point $(3, -7)$ is the center of a circle and the point $(8, 5)$ is on the circle, what is the circumference of the circle?

(A) 13π (B) 15π (C) 18π
(D) 25π (E) 26π
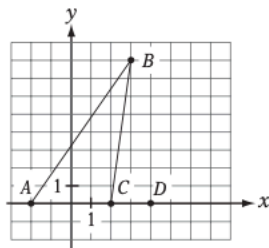
5. In the figure above, point E is to be drawn so that $\triangle CDE$ has the same area as $\triangle ABC$. Which of the following could be the coordinates of E?

(A) $(16, 5)$ (B) $(3, 8)$ (C) $(5, 12)$
(D) $(2, 16)$ (E) $(4, 24)$
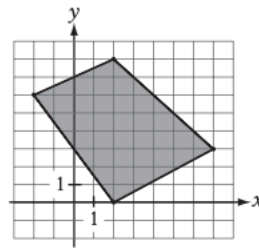
6. What is the area, in square units, of the shaded region in the figure above?

(A) 32
(B) 33
(C) 34
(D) 35
(E) 36
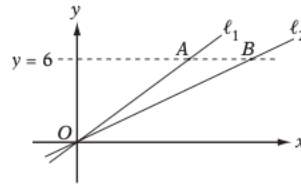
Note: Figure not drawn to scale.

7. Points A and B lie on the line $y = 6$, as shown above. Lines ℓ_1 and ℓ_2 pass through the origin, and ℓ_1 has a slope of $\frac{1}{2}$. If the distance from A to B is 4, what is the slope of line ℓ_2?

10

Jaime is preparing for a bicycle race. His goal is to bicycle an average of at least 280 miles per week for 4 weeks. He bicycled 240 miles the first week, 310 miles the second week, and 320 miles the third week. Which inequality can be used to represent the number of miles, x, Jaime could bicycle on the 4th week to meet his goal?

A) $\dfrac{240 + 310 + 320}{3} + x \geq 280$

B) $240 + 310 + 320 \geq x(280)$

C) $\dfrac{240}{4} + \dfrac{310}{4} + \dfrac{320}{4} + x \geq 280$

D) $240 + 310 + 320 + x \geq 4(280)$

11
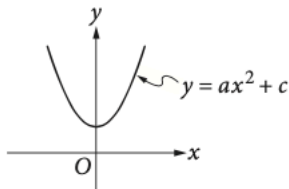
The vertex of the parabola in the xy-plane above is $(0, c)$. Which of the following is true about the parabola with the equation $y = -a(x - b)^2 + c$?

A) The vertex is (b, c) and the graph opens upward.

B) The vertex is (b, c) and the graph opens downward.

C) The vertex is $(-b, c)$ and the graph opens upward.

D) The vertex is $(-b, c)$ and the graph opens downward.

12

Which of the following is equivalent to $\dfrac{4x^2 + 6x}{4x + 2}$?

A) x

B) $x + 4$

C) $x - \dfrac{2}{4x + 2}$

D) $x + 1 - \dfrac{2}{4x + 2}$

13

$$2x^2 - 4x = t$$

In the equation above, t is a constant. If the equation has no real solutions, which of the following could be the value of t ?

A) -3

B) -1

C) 1

D) 3

14

A laundry service is buying detergent and fabric softener from its supplier. The supplier will deliver no more than 300 pounds in a shipment. Each container of detergent weighs 7.35 pounds, and each container of fabric softener weighs 6.2 pounds. The service wants to buy at least twice as many containers of detergent as containers of fabric softener. Let d represent the number of containers of detergent, and let s represent the number of containers of fabric softener, where d and s are nonnegative integers. Which of the following systems of inequalities best represents this situation?

A) $7.35d + 6.2s \leq 300$
   $d \geq 2s$

B) $7.35d + 6.2s \leq 300$
   $2d \geq s$

C) $14.7d + 6.2s \leq 300$
   $d \geq 2s$

D) $14.7d + 6.2s \leq 300$
   $2d \geq s$

15

Which of the following is equivalent to $\left(a + \dfrac{b}{2}\right)^2$?

A) $a^2 + \dfrac{b^2}{2}$

B) $a^2 + \dfrac{b^2}{4}$

C) $a^2 + \dfrac{ab}{2} + \dfrac{b^2}{2}$

D) $a^2 + ab + \dfrac{b^2}{4}$

16

If $a^{\frac{b}{4}} = 16$ for positive integers a and b, what is one possible value of b ?

17

$$\frac{2}{3}t = \frac{5}{2}$$

What value of t is the solution of the equation above?

18
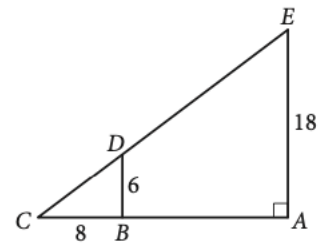
In the figure above, $\overline{BD}$ is parallel to $\overline{AE}$. What is the length of $\overline{CE}$?

19

How many liters of a 25% saline solution must be added to 3 liters of a 10% saline solution to obtain a 15% saline solution?

20

Points A and B lie on a circle with radius 1, and arc $\overparen{AB}$ has length $\dfrac{\pi}{3}$. What fraction of the circumference of the circle is the length of arc $\overparen{AB}$?

10 D

11 B

12 D

13 A

14 A

15 D

16 1, 2, 4, 8, 16

17 15/4, 3.75

18 30

19 3/2, 1.5

20 1/6, .166, .167

11

$$7x + 3y = 8$$
$$6x - 3y = 5$$

For the solution (x, y) to the system of equations above, what is the value of $x - y$?

A) $-\dfrac{4}{3}$

B) $\dfrac{2}{3}$

C) $\dfrac{4}{3}$

D) $\dfrac{22}{3}$

Sunflower Growth

Day	Height (cm)
0	0.00
7	17.93
14	36.36
21	67.76
28	98.10
35	131.00
42	169.50
49	205.50
56	228.30
63	247.10
70	250.50
77	253.80
84	254.50

Sunflower Height over Time
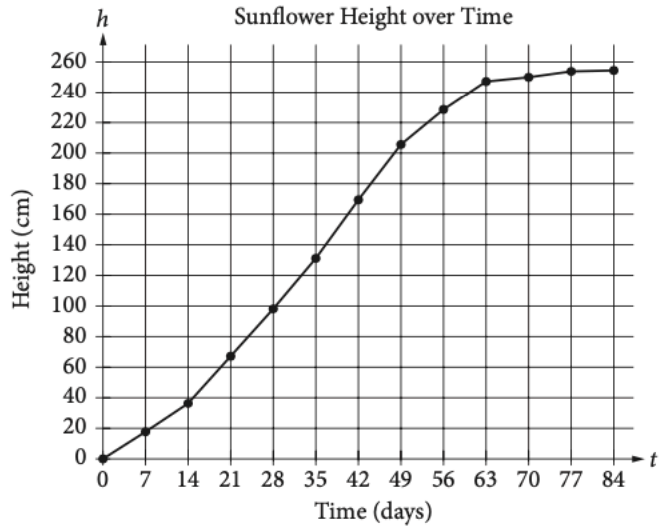
In 1919, H. S. Reed and R. H. Holland published a paper on the growth of sunflowers. Included in the paper were the table and graph above, which show the height h, in centimeters, of a sunflower t days after the sunflower begins to grow.

12

Over which of the following time periods is the average growth rate of the sunflower least?

A) Day 0 to Day 21
B) Day 21 to Day 42
C) Day 42 to Day 63
D) Day 63 to Day 84

13

The function h, defined by $h(t) = at + b$, where a and b are constants, models the height, in centimeters, of the sunflower after t days of growth during a time period in which the growth is approximately linear. What does a represent?

A) The predicted number of centimeters the sunflower grows each day during the period
B) The predicted height, in centimeters, of the sunflower at the beginning of the period
C) The predicted height, in centimeters, of the sunflower at the end of the period
D) The predicted total increase in the height of the sunflower, in centimeters, during the period

The growth rate of the sunflower from day 14 to day 35 is nearly constant. On this interval, which of the following equations best models the height h, in centimeters, of the sunflower t days after it begins to grow?

A) $h = 2.1t - 15$

B) $h = 4.5t - 27$

C) $h = 6.8t - 12$

D) $h = 13.2t - 18$

x	1	2	3	4	5
y	$\frac{11}{4}$	$\frac{25}{4}$	$\frac{39}{4}$	$\frac{53}{4}$	$\frac{67}{4}$

Which of the following equations relates y to x for the values in the table above?

A) $y = \frac{1}{2} \cdot \left(\frac{5}{2}\right)^x$

B) $y = 2 \cdot \left(\frac{3}{4}\right)^x$

C) $y = \frac{3}{4}x + 2$

D) $y = \frac{7}{2}x - \frac{3}{4}$
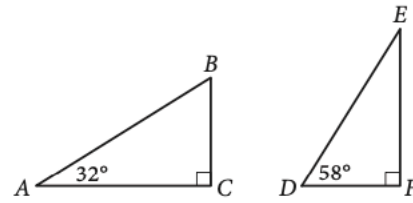
Triangles ABC and DEF are shown above. Which of the following is equal to the ratio $\dfrac{BC}{AB}$?

A) $\dfrac{DE}{DF}$

B) $\dfrac{DF}{DE}$

C) $\dfrac{DF}{EF}$

D) $\dfrac{EF}{DE}$

tread
depth
(d)

riser height
(h)

total rise
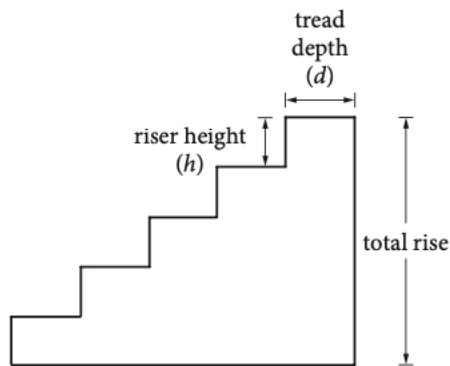
Note: Figure not drawn to scale.

When designing a stairway, an architect can use the riser-tread formula $2h + d = 25$, where h is the riser height, in inches, and d is the tread depth, in inches. For any given stairway, the riser heights are the same and the tread depths are the same for all steps in that stairway.

The number of steps in a stairway is the number of its risers. For example, there are 5 steps in the stairway in the figure above. The total rise of a stairway is the sum of the riser heights as shown in the figure.

17

Which of the following expresses the riser height in terms of the tread depth?

A) $h = \dfrac{1}{2}(25 + d)$

B) $h = \dfrac{1}{2}(25 - d)$

C) $h = -\dfrac{1}{2}(25 + d)$

D) $h = -\dfrac{1}{2}(25 - d)$

18

Some building codes require that, for indoor stairways, the tread depth must be at least 9 inches and the riser height must be at least 5 inches. According to the riser-tread formula, which of the following inequalities represents the set of all possible values for the riser height that meets this code requirement?

A) $0 \le h \le 5$

B) $h \ge 5$

C) $5 \le h \le 8$

D) $8 \le h \le 16$

19

An architect wants to use the riser-tread formula to design a stairway with a total rise of 9 feet, a riser height between 7 and 8 inches, and an odd number of steps. With the architect's constraints, which of the following must be the tread depth, in inches, of the stairway? (1 foot = 12 inches)

A)   7.2

B)   9.5

C)   10.6

D)   15

What is the sum of the solutions to
$(x-6)(x+0.7)=0$?

A) -6.7

B) -5.3

C) 5.3

D) 6.7

A study was done on the weights of different types of fish in a pond. A random sample of fish were caught and marked in order to ensure that none were weighed more than once. The sample contained 150 largemouth bass, of which 30% weighed more than 2 pounds. Which of the following conclusions is best supported by the sample data?

A) The majority of all fish in the pond weigh less than 2 pounds.

B) The average weight of all fish in the pond is approximately 2 pounds.

C) Approximately 30% of all fish in the pond weigh more than 2 pounds.

D) Approximately 30% of all largemouth bass in the pond weigh more than 2 pounds.

Number of States with 10 or More
Electoral Votes in 2008

Electoral votes	Frequency
10	4
11	4
12	1
13	1
15	3
17	1
20	1
21	2
27	1
31	1
34	1
55	1

In 2008, there were 21 states with 10 or more electoral votes, as shown in the table above. Based on the table, what was the median number of electoral votes for the 21 states?

A) 13

B) 15

C) 17

D) 20

23

Height versus Time for a Bouncing Ball
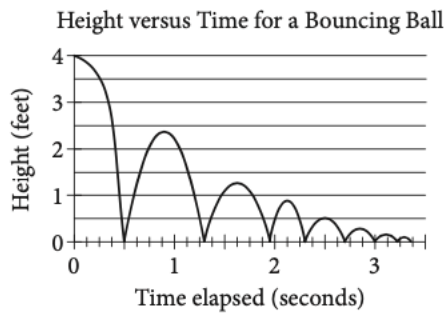
As part of an experiment, a ball was dropped and allowed to bounce repeatedly off the ground until it came to rest. The graph above represents the relationship between the time elapsed after the ball was dropped and the height of the ball above the ground. After it was dropped, how many times was the ball at a height of 2 feet?

A) One

B) Two

C) Three

D) Four

24

A customer's monthly water bill was $75.74. Due to a rate increase, her monthly bill is now $79.86. To the nearest tenth of a percent, by what percent did the amount of the customer's water bill increase?

A) 4.1%

B) 5.1%

C) 5.2%

D) 5.4%

25

x	$f(x)$
0	−2
2	4
6	16

Some values of the linear function f are shown in the table above. What is the value of $f(3)$?

A) 6

B) 7

C) 8

D) 9

A gear ratio $r{:}s$ is the ratio of the number of teeth of two connected gears. The ratio of the number of revolutions per minute (rpm) of two gear wheels is $s{:}r$. In the diagram below, Gear A is turned by a motor. The turning of Gear A causes Gears B and C to turn as well.

Gear A
20 teeth

Gear B
60 teeth
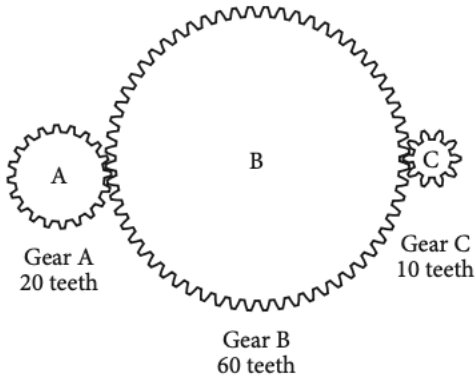
Gear C
10 teeth

If Gear A is rotated by the motor at a rate of 100 rpm, what is the number of revolutions per minute for Gear C?

A)   50

B)   110

C)   200

D) 1,000

In the xy-plane, the graph of $2x^2 - 6x + 2y^2 + 2y = 45$ is a circle. What is the radius of the circle?

A) 5

B) 6.5

C) $\sqrt{40}$

D) $\sqrt{50}$

Two different points on a number line are both 3 units from the point with coordinate -4. The solution to which of the following equations gives the coordinates of both points?

A) $|x + 4| = 3$

B) $|x - 4| = 3$

C) $|x + 3| = 4$

D) $|x - 3| = 4$

29

A motor powers a model car so that after starting from rest, the car travels s inches in t seconds, where $s = 16t\sqrt{t}$. Which of the following gives the average speed of the car, in inches per second, over the first t seconds after it starts?

A) $4\sqrt{t}$

B) $16\sqrt{t}$

C) $\dfrac{16}{\sqrt{t}}$

D) $16t$

30

The scatterplot below shows the amount of electric energy generated, in millions of megawatt-hours, by nuclear sources over a 10-year period.

Electric Energy Generated by Nuclear Sources

Of the following equations, which best models the data in the scatterplot?

A) $y = 1.674x^2 + 19.76x - 745.73$

B) $y = -1.674x^2 - 19.76x - 745.73$

C) $y = 1.674x^2 + 19.76x + 745.73$
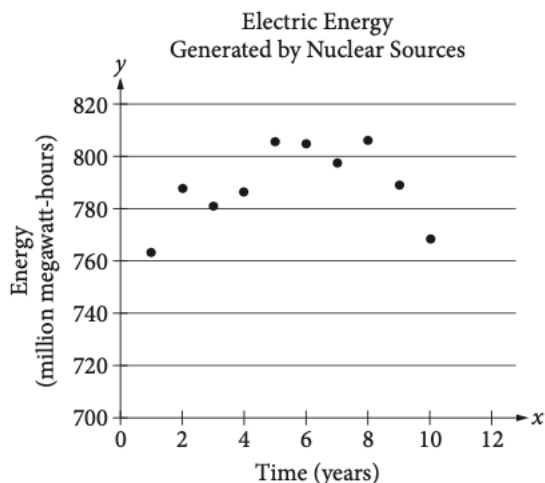
D) $y = -1.674x^2 + 19.76x + 745.73$

31

A group of friends decided to divide the $800 cost of a trip equally among themselves. When two of the friends decided not to go on the trip, those remaining still divided the $800 cost equally, but each friend's share of the cost increased by $20. How many friends were in the group originally?

32

$$2(5x - 20) - (15 + 8x) = 7$$

What value of x satisfies the equation above?

33

A laboratory supply company produces graduated cylinders, each with an internal radius of 2 inches and an internal height between 7.75 inches and 8 inches. What is one possible volume, rounded to the nearest cubic inch, of a graduated cylinder produced by this company?

34

In the xy-plane, the graph of $y = 3x^2 - 14x$ intersects the graph of $y = x$ at the points $(0,0)$ and (a, a). What is the value of a ?

35

The line with the equation $\frac{4}{5}x + \frac{1}{3}y = 1$ is graphed in the xy-plane. What is the x-coordinate of the x-intercept of the line?

36

	Masses (kilograms)					
Andrew	2.4	2.5	3.6	3.1	2.5	2.7
Maria	x	3.1	2.7	2.9	3.3	2.8

Andrew and Maria each collected six rocks, and the masses of the rocks are shown in the table above. The mean of the masses of the rocks Maria collected is 0.1 kilogram greater than the mean of the masses of the rocks Andrew collected. What is the value of x ?

37

Jeremy deposited x dollars in his investment account on January 1, 2001. The amount of money in the account doubled each year until Jeremy had 480 dollars in his investment account on January 1, 2005. What is the value of x ?

38

A school district is forming a committee to discuss plans for the construction of a new high school. Of those invited to join the committee, 15% are parents of students, 45% are teachers from the current high school, 25% are school and district administrators, and the remaining 6 individuals are students. How many more teachers were invited to join the committee than school and district administrators?

11 B	21 D	31 10
12 D	22 B	32 31
13 A	23 C	33 97, 98, 99, 100, 101
14 B	24 D	34 5
15 D	25 B	35 1.25, 5/4
16 B	26 C	36 2.6, 13/5
17 B	27 A	37 30
18 C	28 A	38 8
19 C	29 B	
20 C	30 D	