

Homework answers

Pg 4

1. B 2. D 3. C 4. D 5. 23
6. 5 7. 32.4

1. B

$$\frac{1\frac{3}{4}}{2\frac{1}{2}} = \frac{14}{x}$$

The ratio of $1\frac{3}{4}$ to $2\frac{1}{2}$ is equal to the ratio of 14 to x .

$$1\frac{3}{4} \cdot x = 14 \cdot 2\frac{1}{2}$$

Cross Products

$$\frac{7}{4}x = 14 \cdot \frac{5}{2}$$

Simplify.

$$\frac{7}{4}x = 35$$

Simplify.

$$\frac{4}{7} \cdot \frac{7}{4}x = \frac{4}{7} \cdot 35$$

Multiply each side by $\frac{4}{7}$.

$$x = 20$$

Simplify.

2. D

Let x and y be the two numbers.

$$x + y = 14$$

The sum of two numbers is 14.

$$\frac{x}{y} = -3$$

The ratio of the two numbers is -3 .

$$\frac{x}{y} = -3 \Rightarrow x = -3y$$

$$\begin{array}{ll}
 x + y = 14 & \text{First equation} \\
 (-3y) + y = 14 & \text{Substitute } -3y \text{ for } x. \\
 -2y = 14 & \text{Simplify.} \\
 y = -7 &
 \end{array}$$

Substitute $y = -7$ in the first equation.

$$x + (-7) = 14 \Rightarrow x = 21$$

Therefore the product of the two numbers is

$$x \cdot y = 21 \cdot (-7) = -147.$$

3. C

$$\begin{array}{ll}
 2(x - y) = 3y & \\
 2x - 2y = 3y & \text{Distributive property} \\
 2x = 5y & \text{Add } 2y \text{ to each side.} \\
 \frac{2x}{2} = \frac{5y}{2} & \text{Divide each side by 2.} \\
 x = \frac{5}{2}y & \text{Simplify.} \\
 \frac{x}{y} = \frac{\frac{5}{2}y}{y} & \text{Divide each side by } y. \\
 \frac{x}{y} = \frac{5}{2} & \text{Simplify.}
 \end{array}$$

4. D

Let $6x =$ the length and $7x =$ the width of the rectangle.

$$\begin{array}{ll}
 P = 2\ell + 2w & \text{Perimeter of a rectangle.} \\
 78 = 2(6x) + 2(7x) & P = 78, \ell = 6x, \text{ and } w = 7x \\
 78 = 26x & \text{Simplify.} \\
 3 = x & \text{Divide each side by 26.}
 \end{array}$$

Therefore, the length of the rectangle is $6 \cdot 3$ or 18, and the width of the rectangle is $7 \cdot 3$ or 21. The area of the rectangle is $18 \cdot 21$ or 378.

5. 23

$$\begin{aligned}
 \text{Gas Mileage} &= \frac{\text{Number of Miles Traveled}}{\text{Number of Gallons of Gas Used}} \\
 &= \frac{218.5}{9.5} = 23
 \end{aligned}$$

The car's gas mileage is 23 miles per gallon.

6. 5

$$\begin{aligned}
 \text{Unit Price} &= \frac{\text{Price of Package}}{\text{Number of Units in the Package}} \\
 &= \frac{0.95}{20} = 0.0475
 \end{aligned}$$

The unit price of the vitamin water to the nearest cent is 5.

7. 32.4

$$\begin{aligned}
 \text{Density} &= \frac{\text{mass}}{\text{volume}} \\
 \frac{2.7 \text{ grams}}{1 \text{ cm}^3} &= \frac{x \text{ grams}}{12 \text{ cm}^3}
 \end{aligned}$$

$$x = 2.7 \times 12 = 32.4 \text{ grams}$$

Pg 5

1. D 2. B 3. B 4. 240 5. 8
6. 6 7. $\frac{5}{12}$

1. D

Set up a proportion.

$$\frac{1 \text{ inch}}{5 \text{ miles}} = \frac{10 \text{ inches}}{x \text{ miles}} \Rightarrow x = 50 \text{ miles}$$

$$\frac{1 \text{ inch}}{5 \text{ miles}} = \frac{7.2 \text{ inches}}{y \text{ miles}} \Rightarrow y = 7.2 \times 5 = 36 \text{ miles}$$

The area of the state is 50×36 , or $1,800 \text{ mi}^2$.

2. B

Let the number of students = $27x$ and let the number of teachers = $2x$. Then, there will be $27x + 2x$, or $29x$ students and teachers who are in the meeting. Now set up a proportion.

$$\frac{\text{total in the meeting}}{\text{number of teachers}} = \frac{29x}{2x} \Rightarrow$$

$$\frac{754}{\text{number of teachers}} = \frac{29\cancel{x}}{2\cancel{x}} \Rightarrow$$

$$\text{number of teachers} = \frac{754 \times 2}{29} = 52$$

3. B

Let $5x$ = the volume of cement, $9x$ = the volume of sand, and $13x$ = the volume of gravel. Thus the total volume of concrete is $5x + 9x + 13x$, or $27x$, which is equal to 324 ft^3 .

$$27x = 324 \Rightarrow x = 12$$

Therefore, the amount of cement is $5x = 5 \cdot 12 = 60$.

4. 240

$$1 \text{ hour } 45 \text{ minutes} = 1\frac{3}{4} \text{ hours}$$

Set up a proportion.

$$\frac{84 \text{ miles}}{1\frac{3}{4} \text{ hours}} = \frac{x \text{ miles}}{5 \text{ hours}}$$

$$1\frac{3}{4} \cdot x = 84 \cdot 5 \quad \text{Cross Products}$$

$$\frac{7}{4}x = 420 \quad \text{Simplify.}$$

$$\frac{4}{7} \cdot \frac{7}{4}x = \frac{4}{7} \cdot 420 \quad \text{Multiply each side by } \frac{4}{7}.$$

$$x = 240$$

He can drive 240 miles in 5 hours.

5. 8

Let $2x$ = the number of quarters, $4x$ = the number of dimes, and $7x$ = the number of nickels.

Then the total amount in terms of x ,

$2x(0.25) + 4x(0.1) + 7x(0.05)$, is equal to \$5.00.

$$2x(0.25) + 4x(0.1) + 7x(0.05) = 5.00$$

$$0.5x + 0.4x + 0.35x = 5$$

$$1.25x = 5$$

$$x = 4$$

There are $2x = 2 \cdot 4$, or 8 quarters.

6. 6

$$\frac{5x}{3} = \frac{x+14}{2}$$

$$2(5x) = 3(x+14) \quad \text{Cross Products}$$

$$10x = 3x + 42$$

$$7x = 42$$

$$x = 6$$

7. $\frac{5}{12}$

Let r = the amount of raisin, p = the amount of peanut, and c = the amount of chocolate. Then

$$\frac{r}{p} = \frac{2}{3} \quad \text{The ratio of raisin to peanut is } 2:3.$$

$$3r = 2p \quad \text{Cross Products}$$

$$p = \frac{3}{2}r \quad \text{Solve for } p.$$

$$\frac{p}{c} = \frac{5}{8} \quad \text{The ratio of peanut to chocolate is } 5:8.$$

$$8p = 5c \quad \text{Cross Products}$$

$$p = \frac{5}{8}c \quad \text{Solve for } p.$$

Equate the two equations solved for p .

$$\frac{3}{2}r = \frac{5}{8}c \Rightarrow \frac{2}{3} \cdot \frac{3}{2}r = \frac{2}{3} \cdot \frac{5}{8}c$$

$$\Rightarrow r = \frac{5}{12}c \Rightarrow \frac{r}{c} = \frac{5}{12}$$

- 1.A
- 2.D
- 3.C
- 4.320
- 5.B

Example 1

On Thursday, 240 adults and children attended a show. The ratio of adults to children was 5 to 1. How many children attended the show?

- A) 40
- B) 48
- C) 192
- D) 200

Because the ratio of adults to children was 5 to 1, there were 5 adults for every 1 child. Thus, of every 6 people who attended the show, 5 were adults and 1 was a child. In fractions, $\frac{5}{6}$ of the 240 who attended were adults and $\frac{1}{6}$ were children. Therefore, $\frac{1}{6} \times 240 = 40$ children attended the show, which is choice A.

Ratios on the SAT may be expressed in the form 3 to 1, 3:1, $\frac{3}{1}$, or simply 3.

Example 2

On an architect's drawing of the floor plan for a house, 1 inch represents 3 feet. If a room is represented on the floor plan by a rectangle that has sides of lengths 3.5 inches and 5 inches, what is the actual floor area of the room, in square feet?

- A) 17.5
- B) 51.0
- C) 52.5
- D) 157.5

Because 1 inch represents 3 feet, the actual dimensions of the room are $3 \times 3.5 = 10.5$ feet and $3 \times 5 = 15$ feet. Therefore, the floor area of the room is $10.5 \times 15 = 157.5$ square feet, which is choice D.

Example 3

Scientists estimate that the Pacific Plate, one of Earth's tectonic plates, has moved about 1,060 kilometers in the past 10.3 million years. About how far, in miles, has the Pacific Plate moved during this same time period?

(Use 1 mile = 1.6 kilometers.)

- A) 165
- B) 398
- C) 663
- D) 1,696

Because 1 mile = 1.6 kilometers, the distance is 1,060 kilometers $\times \frac{1 \text{ mile}}{1.6 \text{ kilometers}} = 662.5$ miles, which is about 663 miles. Therefore, the correct answer is choice C.

Example 4

County Y consists of two districts. One district has an area of 30 square miles and a population density of 370 people per square mile, and the other district has an area of 50 square miles and a population density of 290 people per square mile. What is the population density, in people per square mile, for all of County Y?

(Note that this example is a student-produced response question and has no choices. On the SAT, you will grid your answer in the spaces provided on the answer sheet.)

The first district has an area of 30 square miles and a population density of 370 people per square mile, so its total population is

$30 \text{ square miles} \times \frac{370 \text{ people}}{\text{square mile}} = 11,100$ people. The other district has

an area of 50 square miles and a population density of 290 people per square mile, so its total population is $50 \text{ square miles} \times \frac{290 \text{ people}}{\text{square mile}} =$

14,500 people. Thus, County Y has total population $11,100 + 14,500 = 25,600$ people and total area $30 + 50 = 80$ square miles. Therefore, the

population density of County Y is $\frac{25,600}{80} = 320$ people per square mile.

Example 5

A furniture store buys its furniture from a wholesaler. For a particular style of table, the store usually sells a table for 75% more than the cost of the table from the wholesaler. During a sale, the store sells the table for 15% more than the cost from the wholesaler. If the sale price of the table is \$299, what is the usual price for the table?

- A) \$359
- B) \$455
- C) \$479
- D) \$524

The sale price of the table was \$299. This is equal to the cost from the wholesaler plus 15%. Thus, $\$299 = 1.15(\text{cost from the wholesaler})$, and the cost from the wholesaler is $\frac{\$299}{1.15} = \260 . The usual price is the cost from the wholesaler, \$260, plus 75%. Therefore, the usual price the store charges for the table is $1.75 \times \$260 = \455 , which is choice B.

1. C 2. A 3. C 4. C 5. B
6. A 7. A 8. B 9. B 10. D

11. $\frac{25}{4}$ or 6.25 12. 3

1. C

$$1 \text{ m} = 100 \text{ cm}$$

$$1 \text{ m}^3 = (100 \text{ cm})^3 = 1,000,000 \text{ cm}^3$$

$$0.01 \text{ m}^3 = 0.01 \times 1,000,000 \text{ cm}^3 = 10,000 \text{ cm}^3$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$4.54 \text{ grams/cm}^3 = \frac{\text{Mass}}{0.01 \text{ m}^3} = \frac{\text{Mass}}{10,000 \text{ cm}^3}$$

$$\begin{aligned} \text{Mass} &= 4.54 \frac{\text{grams}}{\text{cm}^3} \cdot 10,000 \text{ cm}^3 \\ &= 45,400 \text{ grams} \end{aligned}$$

2. A

$$\begin{aligned} \text{Total time} &= \text{Jason's time} + \text{Donny's time} \\ &= 4 \text{ hour } 25 \text{ min} + 2 \text{ hour } 15 \text{ min} \\ &= 4 \frac{5}{12} \text{ hour} + 2 \frac{1}{4} \text{ hour} = 6 \frac{2}{3} \text{ hour} \end{aligned}$$

The amount Donny received

$$= 1,200 \times \frac{2\frac{1}{4} \text{ hour}}{6\frac{2}{3} \text{ hour}} = 1,200 \cdot \frac{\frac{9}{4}}{\frac{20}{3}} = 1,200 \cdot \frac{9}{4} \cdot \frac{3}{20}$$

$$= 405$$

3. C

If the ratio of white balls to yellow balls is $\frac{3}{10}$,

$3n$ represents the number of white balls and $10n$ represents the number of yellow balls (n is a positive integer).

Since the total number of balls in the bag is $3n + 10n$, or $13n$, and n is a positive integer, the number of balls will be a multiple of 13.

Choice C is correct, because 42 is not a multiple of 13.

4. C

Let m = the number of miles traveled in y minutes. Substitute 60 minutes for 1 hour and set up a proportion.

$$\frac{x}{60} = \frac{m}{y} \quad \leftarrow \begin{array}{l} \text{number of miles} \\ \text{number of minutes} \end{array}$$

$$60m = xy \quad \text{Cross Products}$$

$$m = \frac{xy}{60}$$

5. B

Let x = the number of years it will take the tree to reach a height of 30 feet.

Also, 8 inches = $\frac{8}{12}$ feet.

$$8 + \frac{8}{12}x = 30 \quad \begin{array}{l} \text{The tree is 8 feet tall and will} \\ \text{grow } \frac{8}{12}x \text{ feet in } x \text{ years.} \end{array}$$

$$\frac{8}{12}x = 22$$

$$x = 22 \cdot \frac{12}{8} = 33$$

6. A

m minutes = $60m$ seconds

Let p = the number of pages he reads in $20m$ seconds.

Set up a proportion.

$$\frac{x}{60m} = \frac{p}{20m} \quad \leftarrow \begin{array}{l} \text{number of pages} \\ \text{number of seconds} \end{array}$$

$$60m \cdot p = 20m \cdot x$$

Cross Products

$$p = \frac{20m \cdot x}{60m} = \frac{1}{3}x$$

7. A

$$\frac{x}{y} = 1$$

$$y \cdot \frac{x}{y} = y \cdot 1 \quad \text{Multiply each side by } y.$$

$$x = y \quad \text{Simplify.}$$

$$x - y = y - y \quad \text{Subtract } y \text{ from each side.}$$

$$x - y = 0 \quad \text{Simplify.}$$

$$x - y - 1 = 0 - 1 \quad \text{Subtract 1 from each side.}$$

$$x - y - 1 = -1 \quad \text{Simplify.}$$

8. B

Let m = the number of males in the room and let f = the number of females in the room.

$$\frac{m}{f} = \frac{4}{5} \quad \text{The ratio of males to females is 4 to 5.}$$

$$5m = 4f \quad \text{Cross Products}$$

$$\frac{m+8}{f} = \frac{6}{5} \quad \text{After 8 males enter the room, the ratio of males to females is 6 to 5.}$$

$$5(m+8) = 6f \quad \text{Cross Products}$$

$$5m + 40 = 6f \quad \text{Simplify.}$$

$$4f + 40 = 6f \quad \text{Substitute } 4f \text{ for } 5m.$$

$$40 = 2f \quad \text{Subtract } 2f \text{ from each side.}$$

$$20 = f \quad \text{Divide each side by 2.}$$

Substituting 20 for f in the equation $5m = 4f$ gives $5m = 4 \cdot 20$. Solving for m yields $m = 16$.

The total number of people in the room before the additional males enter the room is

$$m + f = 16 + 20 = 36.$$

9. B

If a person is born every 5 seconds, 12 people are born per minute. If a person dies every 12 seconds, 5 people die per minute. Every minute the population grows by $12 - 5$, or 7, people.

Therefore, it takes $\frac{60}{7}$ seconds, or $8\frac{4}{7}$ seconds, for the population to grow by one person.

10. D

Total area of the wall = $9 \times 12 = 108 \text{ ft}^2$.

Let it take p gallons of paint to paint 108 ft^2 .

Set up a proportion.

$$\frac{1}{s} = \frac{p}{108} \quad \begin{array}{l} \leftarrow \text{number of gallons} \\ \leftarrow \text{number of square feet} \end{array}$$

$$sp = 108 \quad \text{Cross Products}$$

$$p = \frac{108}{s}$$

It takes $\frac{108}{s}$ gallons of paint to paint 108 ft^2 .

Since each gallon costs g dollars, the total cost

will be $\frac{108}{s} \cdot g$ dollars.

11. $\frac{25}{4}$ or 6.25

$$2 \text{ in} = 5 \text{ cm}$$

$$1 \text{ in} = \frac{5}{2} \text{ cm} \quad \text{Divide each side by 2.}$$

$$(1 \text{ in})^2 = \left(\frac{5}{2} \text{ cm}\right)^2 \quad \text{Square both sides.}$$

$$1 \text{ in}^2 = \frac{25}{4} \text{ cm}^2 \quad \text{Simplify.}$$

There are $\frac{25}{4}$ square centimeters in 1 square inch.

12. 3

The reduced length of the painting is $18 - x$ and the reduced width of the painting is $12 - x$.

$$\frac{18 - x}{12 - x} = \frac{5}{3} \quad \text{The new ratio is 5 to 3.}$$

$$3(18 - x) = 5(12 - x) \quad \text{Cross Products}$$

$$54 - 3x = 60 - 5x \quad \text{Distributive Property}$$

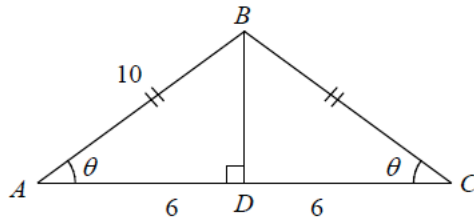
$$54 + 2x = 60 \quad \text{Add } 5x \text{ to each side.}$$

$$2x = 6 \quad \text{Subtract 54 from each side.}$$

$$x = 3 \quad \text{Divide each side by 2.}$$

1. B 2. C 3. B 4. D 5. C

1. B



Draw a perpendicular segment from B to the opposite side AC . Let the perpendicular segment intersect side AC at D . Because the triangle is isosceles, a perpendicular segment from the vertex to the opposite side bisects the base and creates two congruent right triangles.

Therefore, $AD = \frac{1}{2}AC = \frac{1}{2}(12) = 6$.

In right $\triangle ABD$,

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{6}{10} = 0.6.$$

2. C

$$AB^2 = BD^2 + AD^2 \quad \text{Pythagorean Theorem}$$

$$10^2 = BD^2 + 6^2$$

$$100 = BD^2 + 36$$

$$64 = BD^2$$

$$8 = BD$$

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{BD}{AB} = \frac{8}{10} = 0.8$$

3. B

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{BD}{AD} = \frac{8}{6} = \frac{4}{3}$$

4. D

If x and y are acute angles and $\cos x^\circ = \sin y^\circ$,
 $x + y = 90$ by the complementary angle theorem.

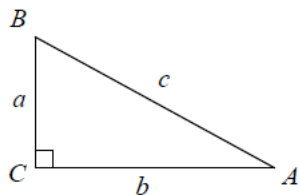
$$(3a - 14) + (50 - a) = 90 \quad x = 3a - 14, \quad y = 50 - a$$

$$2a + 36 = 90 \quad \text{Simplify.}$$

$$2a = 54$$

$$a = 27$$

5. C



$$\text{I. } \sin A = \frac{\text{opposite of } \angle A}{\text{hypotenuse}} = \frac{a}{c}$$

Roman numeral I is true.

$$\text{II. } \cos B = \frac{\text{adjacent of } \angle B}{\text{hypotenuse}} = \frac{a}{c}$$

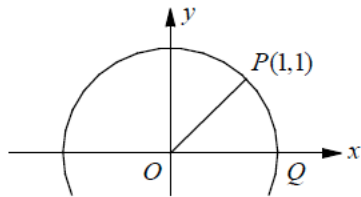
Roman numeral II is true.

$$\text{III. } \tan A = \frac{\text{opposite of } \angle A}{\text{adjacent of } \angle A} = \frac{a}{b}$$

Roman numeral III is false.

1. B 2. C 3. D 4. A

1. B



The graph shows $P(x, y) = P(1, 1)$. Thus, $x = 1$ and $y = 1$. Use the distance formula to find the length of radius OA .

$$OA = \sqrt{x^2 + y^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} \text{ or } \sin \theta = \frac{\sqrt{2}}{2}$$

Therefore, the measure of $\angle POQ$ is 45° ,

which is equal to $45\left(\frac{\pi}{180}\right) = \frac{\pi}{4}$ radians.

Thus, $k = \frac{1}{4}$.

2. C

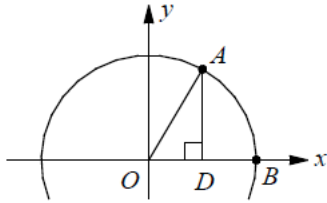
Use the complementary angle theorem.

$$\cos(\theta) = \sin(90^\circ - \theta), \text{ or } \cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right)$$

$$\text{Therefore, } \cos\left(\frac{\pi}{8}\right) = \sin\left(\frac{\pi}{2} - \frac{\pi}{8}\right) = \sin\left(\frac{3\pi}{8}\right).$$

All the other answer choices have values different from $\cos\left(\frac{\pi}{8}\right)$.

3. D

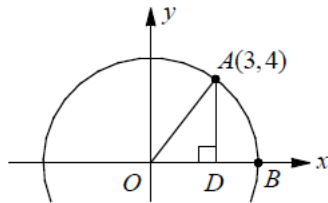


$$\text{In } \triangle OAD, \sin \frac{\pi}{3} = \sin 60^\circ = \frac{AD}{OA} = \frac{AD}{6}.$$

$$\text{Since } \sin 60^\circ = \frac{\sqrt{3}}{2}, \text{ you get } \frac{AD}{6} = \frac{\sqrt{3}}{2}.$$

$$\text{Therefore, } 2AD = 6\sqrt{3} \text{ and } AD = 3\sqrt{3}.$$

4. A



Use the distance formula to find the length of OA .

$$OA = \sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

$$\cos \angle AOD = \frac{OD}{OA} = \frac{3}{5}$$